Section 7 – 4B: Rationalizing the Denominator of a Fraction with a Monomial Term that Contains a Square Root

Each of following factions has a **monomial** expression in the **denominator** that contains a **square root** or a **square root** multiplied by a number.

$\frac{8}{\sqrt{6}}$ $\qquad$ $\frac{10\sqrt{7}}{\sqrt{5}}$ $\qquad$ $\frac{\sqrt{54}}{\sqrt{24}}$ $\qquad$ $\frac{6\sqrt{12}}{8\sqrt{3}}$

It is common to require that the denominator of a fraction not contain any radicals. The process of eliminating radicals from the denominator of a fraction is called **rationalizing the denominator**.

Rationalizing Factions with Monomial terms like $\frac{\sqrt{A}}{\sqrt{B}}$ when A and B have a Common Factor

Many fractions with square roots in both the numerator and denominator can be simplified by reducing the numbers under the square roots. Fractions with only monomial terms that are each under a square root like $\frac{\sqrt{A}}{\sqrt{B}}$ may be **reduced** if A and B have a common factor. If the numbers under the square roots have a common factor then the expressions under the square roots can **be reduced**.

Example 1

Simplify $\frac{\sqrt{12}}{\sqrt{3}}$

$\frac{\sqrt{12}}{\sqrt{3}}$ can also be written as $\sqrt{\frac{12}{3}}$

the 12 and 3 are both factors under a radical sign so they can reduce each other

$= \frac{\sqrt{\cancel{12}^{\,4}}}{\sqrt{\cancel{3}^{\,1}}} = \frac{\sqrt{4}}{\sqrt{1}}$

$= \frac{2}{1} = 2$

Example 2

Simplify $\frac{\sqrt{50}}{\sqrt{8}}$

$\frac{\sqrt{50}}{\sqrt{8}}$ can also be written as $\sqrt{\frac{50}{8}}$

the 50 and 8 are both factors under a radical sign so they can reduce each other

$\frac{\sqrt{\cancel{50}^{\,25}}}{\sqrt{\cancel{8}^{\,4}}} = \frac{\sqrt{25}}{\sqrt{4}}$

$= \frac{5}{2}$

Reducing Factions with Monomial terms like $\frac{C\sqrt{A}}{D\sqrt{B}}$ when A and B and C and D have a Common Factor

Fractions with only monomial terms like $\frac{C\sqrt{A}}{D\sqrt{B}}$ may be **reduced**. If **A and B** have a common factor then the expressions under the square root can be **reduced**. If **C and D** have a common factor then the expressions outside the square root can can be **reduced**.

Warning: An expression under a square root and an expression outside a square root **CAN NOT** reduce each other

Example 3

Simplify: $\frac{10\sqrt{12}}{4\sqrt{27}}$

$\frac{10\sqrt{12}}{4\sqrt{27}}$

the 12 and 27 are both factors under a radical sign so they can reduce each other

the 10 and 4 are both factors outside a radical sign so they can reduce each other

$\frac{\not{10}^{5}\sqrt{\not{12}^{4}}}{\not{4}^{2}\sqrt{\not{27}^{9}}} = \frac{5\sqrt{4}}{2\sqrt{9}}$

$= \frac{5 \bullet 2}{2 \bullet 3} = \frac{5 \bullet \not{2}}{\not{2} \bullet 3} = \frac{5}{3}$

Example 4

Simplify: $\frac{6\sqrt{27}}{9\sqrt{75}}$

$\frac{6\sqrt{27}}{9\sqrt{75}}$

the 27 and 75 are both factors under a radical sign so they can reduce each other

the 6 and 9 are both factors outside a radical sign so they can reduce each other

$\frac{\not{6}^{2}\sqrt{\not{27}^{9}}}{\not{9}^{3}\sqrt{\not{75}^{25}}} = \frac{2\sqrt{9}}{3\sqrt{25}}$

$= \frac{2 \bullet 3}{3 \bullet 5} = \frac{2 \bullet \not{3}}{\not{3} \bullet 5} = \frac{2}{5}$

Rationalizing the Denominator of a Fraction with a Monomial Term that contains a Square Root

Many fractions with square roots do not have a common factor and cannot be simplified by reducing.

$\frac{2}{\sqrt{3}}$ $\frac{7}{\sqrt{11}}$ $\frac{\sqrt{7}}{\sqrt{3}}$ $\frac{2\sqrt{5}}{\sqrt{6}}$

It is common to **require that the denominator not contain any radicals**. In fractions where the expression under the square square roots cannot be reduced to eliminate the square root in the denominator we must find another process that will eliminate the radical from the denominator. The process of eliminating the radical from the denominator of a fraction is called **rationalizing the denominator**.

Multiplying the fraction $\frac{A}{\sqrt{B}}$ **or** $\frac{\sqrt{A}}{\sqrt{B}}$ **by** $\frac{\sqrt{B}}{\sqrt{B}}$ will eliminate the radical from the denominator

$\frac{A}{\sqrt{B}} \bullet \frac{\sqrt{B}}{\sqrt{B}} = \frac{A\sqrt{B}}{B}$ and $\frac{\sqrt{A}}{\sqrt{B}} \bullet \frac{\sqrt{B}}{\sqrt{B}} = \frac{\sqrt{A \bullet B}}{B}$

$\frac{7}{\sqrt{3}}$ multiply the top and bottom by $\sqrt{3}$

$\frac{5}{\sqrt{6}}$ multiply the top and bottom by $\sqrt{6}$

Example 5

Simplify: $\frac{7}{\sqrt{3}}$

$\frac{7}{\sqrt{3}}$ multiply the top and bottom by $\sqrt{3}$

$= \frac{7}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}}$ the $\sqrt{3}$ times itself is 3

$= \frac{7\sqrt{3}}{3}$

Note: The 3 under the radical sign and the 3 outside the radical cannot reduce each other

Example 6

Simplify: $\frac{\sqrt{5}}{\sqrt{6}}$

$\frac{\sqrt{5}}{\sqrt{6}}$ multiply the top and bottom by $\sqrt{6}$

$= \frac{\sqrt{5}}{\sqrt{6}} \bullet \frac{\sqrt{6}}{\sqrt{6}}$ the $\sqrt{6}$ times itself is 6

$= \frac{\sqrt{30}}{6}$

Note: The 30 under the radical sign and the 6 outside the radical cannot reduce each other

Example 7

Simplify: $\frac{6}{\sqrt{10}}$

$\frac{6}{\sqrt{10}}$ multiply the top and bottom by $\sqrt{10}$

$= \frac{6}{\sqrt{10}} \bullet \frac{\sqrt{10}}{\sqrt{10}}$ the $\sqrt{10}$ times itself is 10

$= \frac{6\sqrt{10}}{10}$

the 6 and 10 are both factors outside a radical sign so they can reduce each other

$= \frac{\not{6}^{3}\sqrt{10}}{\not{10}^{5}} = \frac{3\sqrt{10}}{5}$

Example 8

Simplify: $\frac{25}{\sqrt{15}}$

$\frac{25}{\sqrt{15}}$ multiply the top and bottom by $\sqrt{15}$

$= \frac{25}{\sqrt{15}} \bullet \frac{\sqrt{15}}{\sqrt{15}}$ the $\sqrt{15}$ times itself is 15

$= \frac{25\sqrt{10}}{15}$

the 25 and 15 are both factors outside a radical sign so they can reduce each other

$= \frac{\not{25}^{5}\sqrt{10}}{\not{15}^{3}} = \frac{5\sqrt{10}}{3}$

Example 9

Simplify $\frac{3\sqrt{2}}{\sqrt{5}}$

$\frac{3\sqrt{2}}{\sqrt{5}}$ multiply the top and bottom by $\sqrt{5}$

$= \frac{3\sqrt{2}}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}}$ $3\sqrt{2} \bullet \sqrt{5} = 3\sqrt{10}$
the $\sqrt{5}$ times itself is 5

$= \frac{3\sqrt{10}}{5}$

Example 10

Simplify $\frac{5\sqrt{3}}{\sqrt{11}}$

$\frac{5\sqrt{3}}{\sqrt{11}}$ multiply the top and bottom by $\sqrt{11}$

$= \frac{5\sqrt{3}}{\sqrt{11}} \bullet \frac{\sqrt{11}}{\sqrt{11}}$ $5\sqrt{3} \bullet \sqrt{11} = 5\sqrt{33}$
the $\sqrt{11}$ times itself is 11

$= \frac{5\sqrt{33}}{11}$

Example 13

Simplify $\frac{4\sqrt{5}}{\sqrt{6}}$

$\frac{4\sqrt{5}}{\sqrt{6}}$ multiply the top and bottom by $\sqrt{6}$

$= \frac{4\sqrt{5}}{\sqrt{6}} \bullet \frac{\sqrt{6}}{\sqrt{6}}$ $4\sqrt{5} \bullet \sqrt{6} = 4\sqrt{30}$
the $\sqrt{6}$ times itself is 6

$= \frac{4\sqrt{30}}{6}$ reduce $\frac{4}{6}$

$= \frac{2\sqrt{30}}{3}$

Example 14

Simplify $\frac{7\sqrt{10}}{\sqrt{6}}$

the 10 and 6 are both factors under a radical sign so they can reduce each other

$\frac{7\sqrt{5}}{\sqrt{3}}$ multiply the top and bottom by $\sqrt{3}$

$= \frac{7\sqrt{5}}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}}$ $7\sqrt{5} \bullet \sqrt{3} = 7\sqrt{15}$
the $\sqrt{3}$ times itself is 3

$= \frac{7\sqrt{15}}{3}$

