Multiplying a Monomial times a Monomial

To multiply two monomial terms that both contain a term under a radical sign you use the following rule:

$$A\sqrt{B} \bullet C\sqrt{D}$$
$$= A \bullet C \bullet \sqrt{B \bullet D}$$

In other words you multiply the coefficients of the radical terms times the product of the terms under the radicals

Example 1

$\left(5\sqrt{3}\right)\left(4\sqrt{7}\right)$

$= 5 \bullet 4\sqrt{3 \bullet 7}$

$= 20\sqrt{21}$

Example 2

$\left(8\sqrt{5}\right)\left(2\sqrt{7}\right)$

$= 8 \bullet 2\sqrt{5 \bullet 7}$

$= 16\sqrt{35}$

Example 3

$\left(2\sqrt{3}\right)\left(-3\sqrt{5}\right)$

$= 2 \bullet (-3)\sqrt{3 \bullet 5}$

$= -6\sqrt{15}$

After you multiply the monomial terms the term under the radical may be able to be reduced. To reduce the number under the radical you factor it , looking for the largest **perfect square factor** or a **pair of factors** as we did in the first section.

In these examples we will multiply the numbers under the radical together and then reduce by looking for the largest factor that is a perfect square. We then reduce the perfect square factor and simplify

Example 4

$\left(2\sqrt{3}\right)\left(5\sqrt{6}\right) = 2 \bullet 5\sqrt{3 \bullet 6}$
$= 10\sqrt{18}$

reduce $\sqrt{18}$

$= 10\sqrt{9 \bullet 2}$
$= 10\sqrt{9}\sqrt{2}$
$= 10 \bullet 3\sqrt{2}$
$= 30\sqrt{2}$

Example 5

$\left(7\sqrt{6}\right)\left(3\sqrt{8}\right) = 7 \bullet 3\sqrt{6 \bullet 8}$
$= 21\sqrt{48}$

reduce $\sqrt{48}$

$= 21\sqrt{16 \bullet 3}$
$= 21\sqrt{16}\sqrt{3}$
$= 21 \bullet 4\sqrt{2}$
$= 82\sqrt{2}$

Example 6

$\left(3\sqrt{2}\right)\left(5\sqrt{12}\right) = 3 \bullet 5\sqrt{2 \bullet 12}$
$= 15\sqrt{24}$

reduce $\sqrt{24}$

$= 15\sqrt{4 \bullet 6}$
$= 15\sqrt{4}\sqrt{6}$
$= 15 \bullet 2\sqrt{6}$
$= 30\sqrt{6}$

If the numbers under the radicals that are being multiplied are larger it may be faster to list the factors of each number under the radical sign rather then multiplying them together and then factoring them. In these examples we will multiply the numbers under the radical together but instead of listing the product we will just **list all the factors of each number**. This saves the work of getting a large answer that needs to be factored back into the two factors we just multiplied together. We then reduce by taking out pairs of factors or perfect squares.

Example 7

$\left(2\sqrt{7}\right)\left(5\sqrt{14}\right) = 2 \bullet 5\sqrt{7 \bullet 14}$

$= 10\sqrt{7 \bullet 14}$

reduce $\sqrt{7 \bullet 14}$

$= 10\sqrt{7 \bullet 7 \bullet 2}$

$= 10\sqrt{7 \bullet 7}\sqrt{2}$

$= 10 \bullet 7\sqrt{2}$

$= 70\sqrt{2}$

Example 8

$\left(2\sqrt{10}\right)\left(4\sqrt{15}\right) = 2 \bullet 4\sqrt{10 \bullet 15}$

$= 8\sqrt{10 \bullet 15}$

reduce $8\sqrt{10 \bullet 15}$

$= 8\sqrt{2 \bullet 5 \bullet 3 \bullet 5}$

$= 10\sqrt{5 \bullet 5}\sqrt{2 \bullet 3}$

$= 10 \bullet 5\sqrt{3}$

$= 50\sqrt{3}$

Example 9

$\left(\sqrt{6}\right)\left(4\sqrt{18}\right) = 8 \bullet 2\sqrt{6 \bullet 18}$

$= 4\sqrt{6 \bullet 18}$

reduce $4\sqrt{6 \bullet 18}$

$= 4\sqrt{2 \bullet 3 \bullet 2 \bullet 3 \bullet 3}$

$= 4\sqrt{2 \bullet 2}\sqrt{3 \bullet 3}\sqrt{2 \bullet 3}$

$= 4 \bullet 2 \bullet 3\sqrt{3}$

$= 24\sqrt{3}$

Example 10

$\left(\sqrt{20}\right)\left(\sqrt{6}\right) = \sqrt{20 \bullet 6}$

factor the products

$= \sqrt{4 \bullet 5 \bullet 2 \bullet 3}$

$= \sqrt{4}\sqrt{5 \bullet 2 \bullet 3}$

$= 2\sqrt{5 \bullet 2 \bullet 3}$

$= 2\sqrt{30}$

Example 11

$\left(\sqrt{14}\right)\left(\sqrt{21}\right) = \sqrt{14 \bullet 21}$

factor the products

$= \sqrt{2 \bullet 7 \bullet 3 \bullet 7}$

$= \sqrt{7 \bullet 7}\sqrt{2 \bullet 3}$

$= 7\sqrt{2 \bullet 3}$

$= 2\sqrt{6}$

Example 12

$\left(\sqrt{30}\right)\left(\sqrt{50}\right) = \sqrt{30 \bullet 50}$

factor the products

$= \sqrt{3 \bullet 10 \bullet 5 \bullet 10}$

$= \sqrt{10 \bullet 10}\sqrt{3 \bullet 5}$

$= 10\sqrt{3 \bullet 5}$

$= 10\sqrt{15}$

Multiplying a Monomial times a Polynomial (Distributive Property)

The Distributive Rule With Radicals

A distributive problem has a **monomial** term **outside** a parenthesis and a **polynomial** expression **inside**. To distribute you multiply each term inside the parentheses by the term outside the parenthesis.

Example 1

$\sqrt{2}\left(3-\sqrt{5}\right)$

$= 2 \bullet \sqrt{3} - \sqrt{2} \bullet \sqrt{5}$

$= 2\sqrt{3} - \sqrt{10}$

Example 2

$\sqrt{2}\left(4\sqrt{3}-\sqrt{5}\right)$

$= \sqrt{2} \bullet 4\sqrt{3} + \sqrt{2}\sqrt{5}$

$= 4\sqrt{6} + \sqrt{10}$

Example 3

$3\sqrt{5}\left(2-\sqrt{3}\right)$

$= 2 \bullet 3\sqrt{5} - 3\sqrt{5}\sqrt{3}$

$= 6\sqrt{5} - 3\sqrt{15}$

It is common to be able to reduce one or more of the radicals after you distribute.

Example 4

$\sqrt{3}\left(4-5\sqrt{6}\right)$

distribute the $\sqrt{3}$

$= 4\sqrt{3} - 5\sqrt{18}$

factor and reduce $\sqrt{18}$

$= 4\sqrt{3} - 5\sqrt{9 \bullet 2}$

$= 4\sqrt{3} - 5\sqrt{9}\sqrt{2}$

$= 4\sqrt{3} - 15\sqrt{2}$

Example 5

$\sqrt{5}\left(6+2\sqrt{10}\right)$

distribute the $\sqrt{5}$

$= 6\sqrt{5} + 2\sqrt{50}$

factor and reduce $\sqrt{50}$

$= 6\sqrt{5} + 2\sqrt{25 \bullet 2}$

$= 6\sqrt{5} + 2\sqrt{25}\sqrt{2}$

$= 6\sqrt{5} + 10\sqrt{2}$

Example 6

$5\sqrt{2}\left(3\sqrt{6}+7\right)$

distribute the $5\sqrt{2}$

$= 15\sqrt{12} + 35\sqrt{2}$

factor and reduce $\sqrt{12}$

$= 15\sqrt{4 \bullet 3} + 35\sqrt{2}$

$= 15\sqrt{4}\sqrt{3} + 35\sqrt{2}$

$= 30\sqrt{3} + 35\sqrt{2}$

Example 7

$\sqrt{3}\left(\sqrt{3}+\sqrt{15}\right)$

distribute the $\sqrt{3}$

$= \sqrt{9} + 6\sqrt{45}$

reduce $\sqrt{9} = 3$

reduce $\sqrt{45} = \sqrt{9 \bullet 5}$

$= 3 + \sqrt{9 \bullet 5}$

$= 3 + \sqrt{9}\sqrt{5}$

$= 3 + 3\sqrt{5}$

Example 8

$\sqrt{6}\left(5\sqrt{3}+\sqrt{2}\right)$

distribute the $\sqrt{6}$

$= 5\sqrt{18} + \sqrt{12}$

reduce $5\sqrt{18} = 5\sqrt{9 \bullet 2}$

reduce $\sqrt{12} = \sqrt{4 \bullet 3}$

$= 5\sqrt{9}\sqrt{2} + \sqrt{4}\sqrt{3}$

$= 5 \bullet 3\sqrt{2} + 2\sqrt{3}$

$= 15\sqrt{2} + 2\sqrt{3}$

Example 9

$2\sqrt{7}\left(\sqrt{7}+3\sqrt{14}\right)$

distribute the $2\sqrt{7}$

$= 2\sqrt{49} + 6\sqrt{7 \bullet 14}$

reduce $\sqrt{49} = 7$

reduce $6\sqrt{7 \bullet 14} = 6\sqrt{7 \bullet 7 \bullet 2}$

$= 7 + 6 \bullet \sqrt{7 \bullet 7}\sqrt{2}$

$= 7 + 6 \bullet 7\sqrt{2}$

$= 7 + 42\sqrt{2}$

Multiplying a Binomial times a Binomial (FOIL)

We have used the FOIL process in other chapters to find the product of two binomials where the terms of the binomials were variable terms. We will now consider how the product of two binomials will FOIL if the some or all of the terms in the binomials contain radicals.

Multiply $\left(2-\sqrt{5}\right)\left(4-\sqrt{3}\right)$ using FOIL

F. O. I. L.

We distribute both of the terms in the first binomial to both of the terms in the second binomial using the following order

We use the mnemonic **F O I L** to help us remember the process

F The product of the two **First** terms $(2-\sqrt{5})(4-\sqrt{3})=8$

O The product of the two **Outer** terms $(2-\sqrt{5})(4-\sqrt{3})=-2\sqrt{3}$

I The product of the two **Inner** terms $(2-\sqrt{5})(4-\sqrt{3})=-4\sqrt{5}$

L The product of the two Last terms $(2-\sqrt{5})(4-\sqrt{3})=\sqrt{15}$

$(2-\sqrt{5})(4-\sqrt{3})=$

F O I L

$8-2\sqrt{3}-4\sqrt{5}+\sqrt{15}$

It is common to be able to reduce one or more of the radicals after you FOIL. It is also very common to be able to combine Like Terms after you FOIL and reduce the radicals.

Example 1

$(5-\sqrt{3})(2-\sqrt{3})=$

F O I L

$=10-5\sqrt{3}-3\sqrt{3}+\sqrt{9}$

$=10-5\sqrt{3}-3\sqrt{3}+3$

$=13-8\sqrt{3}$

Example 2

$(3-\sqrt{7})(1+\sqrt{7})=$

F O I L

$=3+3\sqrt{7}-1\sqrt{7}+\sqrt{49}$

$=13+3\sqrt{7}-1\sqrt{7}-7$

$=6+2\sqrt{7}$

Example 3

$(3-\sqrt{7})(1+\sqrt{7})=$

F O I L

$=3+3\sqrt{7}-1\sqrt{7}+\sqrt{49}$

$=13+3\sqrt{7}-1\sqrt{7}-7$

$=6+2\sqrt{7}$

Sometime the FOIL process reduces the product to a single integer. This is a very special case. It happens when the two binomials have the terms but one term is an addition and the other term is a subtraction. We call this combination the **Square Root Conjugate**. We will use these **Square Root Conjugates** in the next section.

Example 4

$(5-\sqrt{3})(5+\sqrt{3}) =$

F O I L

$= 25 + 5\sqrt{3} - 5\sqrt{3} - \sqrt{9}$

$= 25 - 3$

$= 22$

Example 5

$(\sqrt{6}+4)(\sqrt{6}-4) =$

F O I L

$= \sqrt{36} - 4\sqrt{6} + 4\sqrt{6} - 16$

$= 6 - 16$

$= -10$

Example 6

$(7-3\sqrt{2})(7+3\sqrt{2}) =$

F O I L

$= 49 + 12\sqrt{2} - 12\sqrt{2} - 9\sqrt{4}$

$= 49 - 18$

$= 31$

Example 7

$(2\sqrt{3}+5)(2\sqrt{3}-5) =$

F O I L

$= 4\sqrt{9} - 10\sqrt{3} + 10\sqrt{3} - 25$

$= 12 - 25$

$= -13$

Example 8

$(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5}) =$

F O I L

$= \sqrt{9} - \sqrt{15} + \sqrt{15} - \sqrt{25}$

$= 3 - 5$

$= -2$

Example 9

$(\sqrt{7}-\sqrt{2})(\sqrt{7}+\sqrt{2}) =$

F O I L

$= \sqrt{49} + \sqrt{14} - \sqrt{14} - \sqrt{4}$

$= 7 - 2$

$= 5$

