Section 7 – 3C: Combining (Adding or Subtracting) Radical Expressions

Like Terms with Radicals

When we combined polynomials in the past we were only able to combine like terms. In a polynomial the like terms were the terms with the exact same variable names. A similar restriction exists for terms that contain a radical. You can only combine radical terms if the radical terms are alike. Two radial terms are alike if the expression under both radicals is **exactly the same** and the power of both radicals is the same.

Example 1

$3\sqrt{5}+4\sqrt{5}$

both radicals are square roots and the
5 under both radicals is exactly the same
so the terms CAN be combined

Example 2

$2\sqrt{3x}-\sqrt{3x}$

both radicals are square roots and the
3x under both radicals is exactly the same
so the terms CAN be combined

Example of radical terms that are NOT alike:

Example 3

$6\sqrt{2}-7\sqrt{3}$

both radicals are square roots but the
$\sqrt{2}$ and the $\sqrt{3}$ under the radical
are NOT exactly the same
so the terms CAN NOT be combined

Example 4

$6\sqrt[3]{2x}-7\sqrt[3]{2xy}$

both radicals are cube roots but the
$2x$ and the $2xy$ under the radical
are NOT exactly the same
so the terms CAN NOT be combined

Example 5

$8\sqrt[3]{2}-4\sqrt[3]{4}$

both radicals are cube roots but the
2 and the 4 under the radical
are NOT exactly the same
so the terms CAN NOT be combined

Example 6

$6\sqrt[3]{5}-\sqrt[2]{5}$

the $\sqrt[3]{5}$ is a cube root and the $\sqrt[2]{5}$ is a squareroot
both radicals are not the same roots
so the terms CAN NOT be combined

Combining Like Terms involving Radicals

Combining like terms involves determining the **total of the coefficients** of all the like terms. You can only combine radical terms if the **radical terms are alike**. Two radial terms are alike if the expression under both radicals is **exactly the same** and the power of both radicals is the same.

To Combine Like Terms involving Radicals

1. Combine the coefficients of the like radical terms by **adding or subtracting the coefficients.**

2. Using the sum of the coefficients as the coefficient of common radical expression.

Example 1

$3\sqrt{5}+4\sqrt{5}$

both radicals are square roots and the $\sqrt{5}$ in both terms are exactly the same so the terms CAN be combined

$3\sqrt{5}+4\sqrt{5}$

combine the 3 and 4

$=7\sqrt{5}$

Example 2

$2\sqrt{3x}-\sqrt{3x}$

both radicals are square roots and the $\sqrt{3x}$ in both terms are exactly the same so the terms CAN be combined

$2\sqrt{3x}-\sqrt{3x}$

combine the 2 and -1

$=\sqrt{3x}$

Example 3

$8\sqrt[3]{2}-4\sqrt[3]{2}$

both radicals are cube roots and the the $\sqrt[3]{2}$ in both terms are exactly the same so the terms CAN be combined

$8\sqrt[3]{2}-4\sqrt[3]{2}$

combine the 8 and -4

$=4\sqrt[3]{2}$

Example 4

$2\sqrt[3]{5y^2}-5\sqrt[3]{5y^2}$

both radicals are cube roots and the the $\sqrt[3]{5y^2}$ in both terms are exactly the same so the terms CAN be combined

$2\sqrt[3]{5y^2}-5\sqrt[3]{5y^2}$

combine the 2 and -5

$=-3\sqrt[3]{5y^2}$

Example 5

$4\sqrt{3}+2\sqrt{3}-9\sqrt{3}$

all three are square roots of 3

combine the 4, 2 and -9

$=-3\sqrt{3}$

Example 6

$-6\sqrt[3]{7x}-\sqrt[3]{7x}+3\sqrt[3]{7x}$

all three are cube roots of 7x

combine the -6, -1 and 3

$=-4\sqrt[3]{7}$

Sometimes only some of the terms in an expression are alike. In that case, then you can only combine the terms that are alike.

Example 7

$3\sqrt{6}-7\sqrt{2}+5\sqrt{6}$

only the $3\sqrt{6}$ and $5\sqrt{6}$ are like terms
combine only the 3 and the 5

$=8\sqrt{6}-7\sqrt{2}$

Example 8

$-2\sqrt[3]{11}+6\sqrt[3]{2}-3\sqrt[3]{11}$

only the $-2\sqrt[3]{11}$ and $-3\sqrt[3]{11}$ are like terms
combine only the -2 and the -3

$=-5\sqrt[3]{11}+6\sqrt[3]{2}$

Example 9

$2\sqrt{3}+8\sqrt{2}-9\sqrt{3}-5\sqrt{2}$

the $2\sqrt{3}$ and $-9\sqrt{3}$ are like terms
combine the 2 and the -9
$-7\sqrt{3}$

the $+8\sqrt{2}$ and $-5\sqrt{2}$ are like terms
combine the 8 and the -5
$3\sqrt{2}$

$=-7\sqrt{3}-3\sqrt{2}$

Example 10

$7\sqrt[3]{11}+3\sqrt{11}-9\sqrt[3]{11}-10\sqrt{11}$

the $7\sqrt[3]{11}$ and $-9\sqrt[3]{11}$ are like terms
combine the 7 and the -9
$-2\sqrt[3]{11}$

the $+3\sqrt{11}$ and $-10\sqrt{11}$ are like terms
combine the 3 and the -10
$-7\sqrt{11}$

$=-2\sqrt[3]{11}-7\sqrt{11}$

Simplifying Radial Expressions and then Combining Like Radical Terms

Some radical expressions do not appear to have like radical terms but after the radicals are simplified there may be like radical terms that can now be combined.

Example 11

$5\sqrt{7}-\sqrt{28}$

$\sqrt{7}$ and $\sqrt{28}$ are NOT like terms
but the $\sqrt{28}$ can be reduced

$-\sqrt{28}=-\sqrt{4\bullet 7}=-2\sqrt{7}$

so $5\sqrt{7}-\sqrt{28}$
$=5\sqrt{7}-2\sqrt{7}$
which can be combined

$=3\sqrt{7}$

Example 12

$5\sqrt[3]{3}-6\sqrt[3]{24}$

$\sqrt[3]{3}$ and $\sqrt[3]{24}$ are NOT like terms
but the $\sqrt[3]{24}$ can be reduced

$-6\sqrt[3]{24}=-6\sqrt[3]{8\bullet 3}=-6(2)\sqrt[3]{3}=-12\sqrt[3]{3}$

so $5\sqrt[3]{3}-6\sqrt[3]{24}$
$=5\sqrt[3]{3}-12\sqrt[3]{3}$
which can be combined

$=-7\sqrt[3]{3}$

Example 13

$8\sqrt{20}-\sqrt{45}$

$8\sqrt{20}=8\sqrt{4\bullet 5}=8(2)\sqrt{5}=16\sqrt{5}$

$-\sqrt{45}=-\sqrt{9\bullet 5}=-(3)\sqrt{5}=-3\sqrt{5}$

so $8\sqrt{20}-\sqrt{45}$
$=16\sqrt{5}-3\sqrt{5}$
which can be combined

$=13\sqrt{5}$

Example 14

$-4\sqrt[3]{16}+5\sqrt[3]{2}$

$\sqrt[3]{16}$ and $\sqrt[3]{2}$ are NOT like terms
but the $-2\sqrt[3]{16}$ can be reduced

$-4\sqrt[3]{16}=-4\sqrt[3]{8\bullet 2}=-4(2)\sqrt[3]{2}$

so $-4\sqrt[3]{16}+5\sqrt[3]{2}$
$=-8\sqrt[3]{2}+5\sqrt[3]{2}$
which can be combined

$=-3\sqrt[3]{2}$

Sometimes even after you simplify each radical term only some of the terms in an expression are alike. In that case you can only combine the terms that are alike.

Example 15

$2\sqrt{27} - \sqrt{50} + 5\sqrt{12}$

simplify:

$2\sqrt{27} = 2\sqrt{9 \bullet 3} = 2(3)\sqrt{3} = 6\sqrt{3}$

$-\sqrt{50} = -\sqrt{25 \bullet 2} = -(5)\sqrt{2} = -5\sqrt{2}$

$+5\sqrt{12} = 5\sqrt{4 \bullet 3} = 5(2)\sqrt{3} = 10\sqrt{3}$

combine like terms

$= 6\sqrt{3} - 5\sqrt{2} + 10\sqrt{3}$

$= 16\sqrt{3} - 5\sqrt{2}$

Example 16

$4\sqrt[3]{32} + 3\sqrt[3]{5} - \sqrt[3]{108}$

simplify:

$4\sqrt[3]{32} = 4\sqrt[3]{8 \bullet 4} = 4(2)\sqrt[3]{4} = 8\sqrt[3]{4}$

$+3\sqrt[3]{12}$ is simplified

$-\sqrt[3]{108} = -\sqrt[3]{27 \bullet 4} = -(3)\sqrt[3]{4} = -3\sqrt[3]{4}$

combine like terms

$= 8\sqrt[3]{4} + 3\sqrt[3]{12} - 3\sqrt[3]{4}$

$= 5\sqrt[3]{4} + 3\sqrt[3]{12}$

