Section 7 – 3B: Simplifying Radical Expressions

Simplifying Square Root Radicals with Variable Factors

The last section introduced the concept of reducing radials by taking out pairs of a common factor. A pair of the same factor under a square root form a perfect square. This means that if you have a pair of the same factors under a square root they can be reduced to a rational number.

$\sqrt{4} = \sqrt{2 \bullet 2} = 2$
a pair of 2's under
a square root reduce to
the whole number 2

$\sqrt{9} = \sqrt{3 \bullet 3} = 3$
a pair of 3's under
a square root reduce to
the whole number 3

$\sqrt{81} = \sqrt{3 \bullet 3 \bullet 3 \bullet 3}$
$= \sqrt{3 \bullet 3} \bullet \sqrt{3 \bullet 3}$
$= 3 \bullet 3 = 9$
2 pair of 3's under
a square root reduce to
the whole number 9

This fact also allows us to reduce radical expressions with terms that contain variables as factors under the square root symbol. Completely factor the expression under the square root into its many factors and then take out the pairs of same factors.

Assume that all variables represent positive numbers.

$\sqrt{a^2} = \sqrt{a \bullet a} = a$
where a is a positive number

Example 1

$\sqrt{a^7}$
completly factor a^7

$= \sqrt{a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a}$
put pairs of the same factor
under their own square root

$= \sqrt{a \bullet a} \bullet \sqrt{a \bullet a} \bullet \sqrt{a \bullet a} \bullet \sqrt{a}$
each of the 3 pairs of a's
can be reduced to an a

$= a \bullet a \bullet a \bullet \sqrt{a}$

$= a^3\sqrt{a}$

Example 2

$\sqrt{a^6}$
completly factor a^6

$= \sqrt{a \bullet a \bullet a \bullet a \bullet a \bullet a}$
put pairs of the same factor
under their own square root

$= \sqrt{a \bullet a} \bullet \sqrt{a \bullet a} \bullet \sqrt{a \bullet a}$
each of the 2 pairs of a's
can be reduced to an a

$= a \bullet a \bullet a$

$= a^3$

Save Time by Determining the Number Of Pairs

We can save some work by determining **how many pairs of factors a variable under the square root symbol has.** Write each pair under its own square root. Reduce each of these square roots. If the exponent of the variable under the square is **odd** then a single power of that variable will remain under the square root. If the exponent of the variable is **even** then that variable will not appear under the square root.

Example 1

Simplify: $\sqrt{y^6}$

y^6 has 3 pairs of y^2
with none left over so

$\sqrt{y^6} = \sqrt{y^2} \bullet \sqrt{y^2} \bullet \sqrt{y^2}$

$\sqrt{y^6} = y \bullet y \bullet y$

$\sqrt{y^6} = y^3$

Example 2

$\sqrt{x^7}$

x^7 has 3 pairs of x^2
with 1 x left over so

$\sqrt{x^7} = \sqrt{x^2} \bullet \sqrt{x^2} \bullet \sqrt{x^2} \bullet \sqrt{x}$

$\sqrt{x^7} = x \bullet x \bullet x \bullet \sqrt{x}$

$\sqrt{x^7} = x^3\sqrt{x}$

Example 3

$\sqrt{x^8}$

x^8 has 4 pairs of x^2
with none left over so

$\sqrt{x^8} = \sqrt{x^2} \bullet \sqrt{x^2} \bullet \sqrt{x^2} \bullet \sqrt{x^2}$

$\sqrt{x^8} = x \bullet x \bullet x \bullet x$

$\sqrt{x^8} = x^4$

We can eliminate writing the separate pairs of square roots and simplify each variable in one step by asking how many pairs does the variable have and is one left over or not.

Example 4

Simplify: $\sqrt{y^7}$

y^7 has 3 pairs of y^2
with 1 y left over

$\sqrt{y^7} = y^3\sqrt{y}$

Example 5

Simplify: $\sqrt{x^6}$

x^6 has 3 pairs of x^2
with no x left over

$\sqrt{x^6} = x^3$

Example 6

Simplify: $\sqrt{w^{11}}$

w^{11} has 5 pairs of w^2
with 1 w left over

$\sqrt{w^{11}} = w^5\sqrt{w}$

Example 7

Simplify: $\sqrt{x^8 y^7}$

x^8 has 5 pairs of x^2
with none left over

$\sqrt{x^8} = x^4$

y^7 has 3 pairs of y^2
with 1 y left over

$\sqrt{y^7} = y^3\sqrt{y}$

$\sqrt{x^8 y^7} = x^4 y^3\sqrt{y}$

Example 8

Simplify: $\sqrt{x^{11} y^6}$

x^{11} has 5 pairs of x^2
with 1 x left over

$\sqrt{x^{11}} = x^5\sqrt{x}$

y^6 has 3 pairs of y^2
with none left over

$\sqrt{y^6} = y^3$

$\sqrt{x^{11} y^6} = x^5 y^3\sqrt{x}$

Example 9

Simplify: $\sqrt{x^2 y^5 w^9}$

1 pair of x, 1 left over

$\sqrt{x^2} = x$

2 pair of y, 1 left over

$\sqrt{y^5} = y^2\sqrt{y}$

4 pair of w, 1 left over

$\sqrt{w^9} = w\sqrt{w}$

$\sqrt{x^2 y^5 w^9} = xy^2 w^4\sqrt{yw}$

Example 10

$\sqrt{x^2 y^5 w^3}$

$\sqrt{x^2} = x$

$\sqrt{y^5} = y^2\sqrt{y}$

$\sqrt{w^3} = w\sqrt{w}$

$\sqrt{x^2 y^5 w^3} = xy^2 w\sqrt{yw}$

Example 11

$\sqrt{x^{12} y^8 w^7}$

$\sqrt{x^{12}} = x^6$

$\sqrt{y^8} = y^4$

$\sqrt{w^7} = w^3\sqrt{w}$

$\sqrt{x^{12} y^8 w^7} = x^6 y^4 w^3\sqrt{w}$

Example 12

$\sqrt{x^5 y^6 w^8}$

$\sqrt{x^5} = x^2\sqrt{x}$

$\sqrt{y^6} = y^3$

$\sqrt{w^8} = w^4$

$\sqrt{x^5 y^6 w^8} = x^2 y^3 w^4\sqrt{x}$

Example 13

$\sqrt{x^5 y^7 w^3}$

$\sqrt{x^5} = x^2\sqrt{x}$

$\sqrt{y^7} = y^3\sqrt{y}$

$\sqrt{w^3} = w\sqrt{w}$

$\sqrt{x^5 y^7 w^3} = x^2 y^3 w\sqrt{xyw}$

t is common that the factors under the square root will contain **the product of a number and one or more variables.** To reduce these radicals reduce the coefficient of the variables by looking for perfect square factors or pairs of factors. Then reduce each variable by asking how many pairs of that variable there are and is one left over that will remain under the square root.

Example 1

$\sqrt{12x^5y^6}$

$\sqrt{12} = \sqrt{4 \bullet 3} = 2\sqrt{3}$

$\sqrt{x^5} = x^2\sqrt{x}$

$\sqrt{y^6} = y^3$

$\sqrt{12x^5y^2} = 2x^2y^3\sqrt{3x}$

Example 2

$\sqrt{18x^8y^{10}}$

$\sqrt{18} = \sqrt{9 \bullet 2} = 3\sqrt{2}$

$\sqrt{x^8} = x^4$

$\sqrt{y^{10}} = y^5$

$\sqrt{18x^8y^{10}} = 3x^4y^5\sqrt{2}$

Example 3

$\sqrt{36x^4y^3}$

$\sqrt{36} = 6$

$\sqrt{x^4} = x^2$

$\sqrt{y^3} = y\sqrt{y}$

$\sqrt{36x^4y^3} = 6x^2y\sqrt{y}$

Example 4

$\sqrt{44x^4y^9}$

$\sqrt{44} = \sqrt{4 \bullet 11} = 2\sqrt{11}$

$\sqrt{x^4} = x^2$

$\sqrt{y^9} = y^4\sqrt{y}$

$\sqrt{44x^4y^9} = 2x^2y^4\sqrt{11y}$

Example 5

$\sqrt{49x^2y^{12}}$

$\sqrt{49} = 7$

$\sqrt{x^2} = x$

$\sqrt{y^{12}} = y^6$

$\sqrt{49x^2y^{12}} = 7xy^6$

Example 6

$\sqrt{63x^3y^{11}}$

$\sqrt{63} = \sqrt{9 \bullet 7} = 3\sqrt{7}$

$\sqrt{x^3} = x\sqrt{x}$

$\sqrt{y^{11}} = y^5\sqrt{y}$

$\sqrt{63x^3y^{11}} = 3xy^5\sqrt{7xy}$

Simplifying Cube Root Radicals with Variable Factors

The last section introduced the concept of reducing radials with variable factors by taking out pairs of a common factor. Three of the same variable factors under a cube root form a **perfect cube.**

$\sqrt[3]{x^3} = x$ $\sqrt{y^3} = y$

This fact allows us to reduce radical expressions that have variables as factors under the **cube root symbol.** Assume that all variables represent positive numbers.

Example 1

Simplify: $\sqrt[3]{x^6}$

x^6 has 2 groups of x^3 with NO x left over

$\sqrt[3]{x^6} = \sqrt[3]{x^3} \bullet \sqrt[3]{x^3}$

$= x \bullet x$

$= x^2$

Example 2

Simplify: $\sqrt[3]{x^7}$

x^7 has 2 groups of x^3 with 1 x left over

$\sqrt[3]{x^7} = \sqrt[3]{x^3} \bullet \sqrt[3]{x^3} \bullet \sqrt[3]{x}$

$= x \bullet x \bullet \sqrt[3]{x}$

$= x^2 \bullet \sqrt[3]{x}$

Example 3

Simplify: $\sqrt[3]{x^{11}}$

x^{11} has 3 groups of x^3 with x^2 left over

$\sqrt[3]{x^{11}} = \sqrt[3]{x^3} \bullet \sqrt[3]{x^3} \bullet \sqrt[3]{x^3} \bullet \sqrt[3]{x^2}$

$= x \bullet x \bullet x \bullet \sqrt[3]{x^2}$

$= x^3 \bullet \sqrt[3]{x^2}$

Example 4

Simplify: $\sqrt[3]{x^{12}}$

x^{12} has 4 groups of x^3 with no x's left over

$\sqrt[3]{x^{12}} = x^4$

Example 5

Simplify: $\sqrt[3]{y^{10}}$

y^{10} has 3 groups of y^3 with y left over

$\sqrt[3]{y^{10}} = y^3 \bullet \sqrt[3]{y}$

Example 6

Simplify: $\sqrt[3]{w^8}$

w^8 has 2 groups of w^3 with w^2 left over

$\sqrt[3]{w^8} = w^2 \bullet \sqrt[3]{w^2}$

Example 7

Simplify: $\sqrt[3]{x^8 y^{10}}$

x^8 has 2 groups of x^3
with x^2 left over
$\sqrt[3]{x^8} = x^2 \bullet \sqrt[3]{x^2}$

y^{10} has 3 groups of x^3
with 1 y left over
$\sqrt[3]{y^{10}} = y^3 \bullet \sqrt[3]{y}$

$\sqrt[3]{x^8 y^{10}} = x^2 y^3 \bullet \sqrt[3]{x^2 y}$

Example 8

Simplify: $\sqrt[3]{x^{12} y^{13}}$

x^{12} has 4 groups of x^3
with no x left over
$\sqrt[3]{x^{12}} = x^4$

y^{13} has 4 groups of x^3
with 1 y left over
$\sqrt[3]{y^{13}} = y^4 \bullet \sqrt[3]{y}$

$\sqrt[3]{x^{12} y^{13}} = x^4 y^4 \bullet \sqrt[3]{y}$

Example 9

Simplify: $\sqrt[3]{x^5 y^{14}}$

x^5 has 1 group of x^3
with x^2 left over
$\sqrt[3]{x^5} = x \bullet \sqrt[3]{x^2}$

y^{14} has 4 groups of x^3's
with y^2 left over
$\sqrt[3]{y^{14}} = y^4 \bullet \sqrt[3]{y^2}$

$\sqrt[3]{x^5 y^{14}} = xy^4 \bullet \sqrt[3]{x^2 y^2}$

Putting it all together

Example 10

Simplify: $\sqrt[3]{24x^6 y^4}$

$\sqrt[3]{24} = \sqrt[3]{8 \bullet 3} = 2 \bullet \sqrt[3]{3}$
$\sqrt[3]{x^6} = x^3$
$\sqrt[3]{y^4} = y \bullet \sqrt[3]{y}$

$\sqrt[3]{24x^6 y^4} = 4x^2 y \bullet \sqrt[3]{y}$

Example 11

Simplify: $\sqrt[3]{54x^8 y^{12}}$

$\sqrt[3]{54} = \sqrt[3]{27 \bullet 2} = 3 \bullet \sqrt[3]{2}$
$\sqrt[3]{x^8} = x^2 \bullet \sqrt[3]{x^2}$
$\sqrt[3]{y^{12}} = y^4$

$\sqrt[3]{54x^8 y^5} = 3x^2 y^4 \bullet \sqrt[3]{2x^2}$

Example 12

Simplify: $\sqrt[3]{-128x^{15} y^{10}}$

$\sqrt[3]{-128} = \sqrt[3]{-64 \bullet 2} = -4 \bullet \sqrt[3]{2}$
$\sqrt[3]{x^{15}} = x^5$
$\sqrt[3]{y^{10}} = y^3 \bullet \sqrt[3]{y}$

$\sqrt[3]{-128x^{15} y^{10}} = -4x^5 y^3 \bullet \sqrt[3]{2y}$

