Section 7 – 3A: **Simplifying Radical Expressions**

When you find the square root of a perfect square there will not be a square root in the answer.

$\sqrt{9} = 3$ $\sqrt{25} = 5$ $\sqrt{\frac{36}{49}} = \frac{6}{7}$

Most of the time the number under the square root is not a perfect square. The square root of any number that is not a perfect square **CANNOT** be replaced by any fraction, decimal or whole number. **This type of number is an irrational number.** It represents **a decimal that never ends or repeats**. Since you cannot write such a decimal that never ends or repeats you cannot state such a square root as a number without a radical sign.

$\sqrt{7}$ cannot be replaced with a decimal or fraction, it stays $\sqrt{7}$	$\sqrt{21}$ cannot be replaced with a decimal or fraction, it stays $\sqrt{21}$	$\sqrt{11}$ cannot be replaced with a decimal or fraction, it stays $\sqrt{11}$

Some square roots can be reduced to the product of a rational number times the square root of a smaller number

If the number under the square root is not a perfect square then it cannot replaced with a number without a radical sign. It may be replaced with an expression that has a smaller number under the radical sign.

$\sqrt{8}$ can be replaced by $2\sqrt{2}$	$\sqrt{12}$ can be replaced by $2\sqrt{3}$	$\sqrt{75}$ can be replaced by $5\sqrt{3}$

We call the process of replacing one square root expression with another square root expression that has a smaller number under the radical sign reducing or **simplifying the square root**.

Multiplication Rule for Square Roots

Assume all variables are positive numbers

If the number C has factors A and B then

$$\sqrt{C} = \sqrt{A \bullet B} = \sqrt{A} \bullet \sqrt{B}$$

This rule allows you to factor a number under the square root into two (or more) factors and write the factors as a product under two separate square roots. If one of the factors is a perfect square then that square root can be reduced leaving you with a number outside the radical times a smaller square root then the original square root you started with.

Simplifying Square Root Radicals using PERFECT SQUARE FACTORS

This reducing technique requires that you find the largest perfect square that is a factor of the original radicand. State the number under the square root as the product the **largest perfect square factor** and a second factor. Break the square root expression into the product of the two factors and then reduce the perfect square term.

Example 1

Simplify: $\sqrt{18}$

$= \sqrt{9 \bullet 2}$

$= \sqrt{9} \bullet \sqrt{2}$

$= 3\sqrt{2}$

Example 2

Simplify: $\sqrt{12}$

$= \sqrt{4 \bullet 3}$

$= \sqrt{4} \bullet \sqrt{3}$

$= 2\sqrt{3}$

Example 3

Simplify: $\sqrt{24}$

$= \sqrt{4 \bullet 6}$

$= \sqrt{4} \bullet \sqrt{6}$

$= 2\sqrt{6}$

Example 1 Note: To use this technique you must factor $\sqrt{18}$ as $\sqrt{9 \bullet 2}$ and not as $\sqrt{3 \bullet 6}$
The factor must be the largest perfect square root factor

Example 4

simplify: $\sqrt{40}$

$= \sqrt{4 \bullet 10}$

$= \sqrt{4} \bullet \sqrt{10}$

$= 2\sqrt{10}$

Example 5

Simplify: $\sqrt{48}$

$= \sqrt{16 \bullet 3}$

$= \sqrt{16} \bullet \sqrt{3}$

$= 4\sqrt{3}$

Example 6

$\sqrt{72}$

$= \sqrt{36 \bullet 2}$

$= \sqrt{36} \bullet \sqrt{2}$

$= 6\sqrt{2}$

Example 6 Note: To use this technique you must factor $\sqrt{72}$ as $\sqrt{36 \bullet 2}$ and not as $\sqrt{9 \bullet 8}$
The factor must be the largest perfect square root factor

Simplifying Square Root Radicals using PAIRS OF FACTORS

If you can find the largest perfect square factor of the radicand then reducing the radical expression is a short process. Many students cannot find the largest perfect square factor or they do not want to take the extended time this may take. There is a alternate approach that is favored by many students.

Two of the same factors under a square root form a perfect square. This means that if you have a pair of the same factors under a square root they can be reduced to a rational number.

$\sqrt{4} = \sqrt{2 \bullet 2} = 2$

a pair of 2's under
a square root reduce to
the whole number 2

$\sqrt{9} = \sqrt{3 \bullet 3} = 3$

a pair of 3's under
a square root reduce to
the whole number 3

$\sqrt{25} = \sqrt{5 \bullet 5} = 5$

a pair of 5's under
a square root reduce to
the whole number 5

This fact allows us to use the Multiplication Rule for Square Roots to completely factor a radicand into its many factors and then take out t**he pairs of same factors.**

Example 1

$\sqrt{24}$

completly factor 24

$= \sqrt{2 \bullet 2 \bullet 2 \bullet 3}$

put pairs of the same factor
under thier own square root

$= \sqrt{2 \bullet 2} \bullet \sqrt{2 \bullet 3}$

the pair of 2's can reduced

$= 2 \bullet \sqrt{6}$

Example 2

$\sqrt{54}$

completly factor 54

$= \sqrt{3 \bullet 3 \bullet 3 \bullet 2}$

put pairs of the same factor
under thier own square root

$= \sqrt{3 \bullet 3} \bullet \sqrt{3 \bullet 2}$

the pair of 2's can reduced

$= 3 \bullet \sqrt{6}$

Example 3

$\sqrt{48}$

completly factor 48

$= \sqrt{2 \bullet 2 \bullet 2 \bullet 2 \bullet 3}$

put pairs of the same factor
under thier own square root

$= \sqrt{2 \bullet 2} \bullet \sqrt{2 \bullet 2} \bullet \sqrt{3}$

each pair of 2's can reduced

$= 2 \bullet 2 \bullet \sqrt{3} = 4\sqrt{3}$

Example 4

$\sqrt{50}$

$= \sqrt{5 \bullet 10}$

$= \sqrt{5 \bullet 2 \bullet 5}$

$= \sqrt{5 \bullet 5} \bullet \sqrt{2}$

$= 5\sqrt{2}$

Example 5

$\sqrt{80}$

$= \sqrt{8 \bullet 10}$

$\sqrt{2 \bullet 2 \bullet 2 \bullet 2 \bullet 5}$

$= \sqrt{2 \bullet 2} \bullet \sqrt{2 \bullet 2} \bullet \sqrt{5}$

$= 2 \bullet 2\sqrt{5}$

$= 4\sqrt{5}$

Example 6

$\sqrt{120}$

$= \sqrt{12 \bullet 10}$

$= \sqrt{2 \bullet 6 \bullet 2 \bullet 5}$

$= \sqrt{2 \bullet 2 \bullet 3 \bullet 2 \bullet 5}$

$= \sqrt{2 \bullet 2} \quad \sqrt{3 \bullet 2 \bullet 5}$

$= 2\sqrt{30}$

Finding the Cube Root of a Perfect Cube

The radical sign $\sqrt[3]{}$ is called a **cube root sign**.

$\sqrt[3]{a}$ is read as "the cube root of a"

$\sqrt[3]{a}$ asks you "What number used as a factor 3 times is equal to a?

Example 1

$\sqrt[3]{8}$ asks "What number used as a factor 3 times is equal to 8?

$$\sqrt[3]{8} = \sqrt[3]{2 \bullet 2 \bullet 2} = 2$$

a group of three 2's under a cube root reduce to the whole number 2

Example 2

$\sqrt[3]{27}$ asks "What number used as a factor 3 times is equal to 27?

$$\sqrt[3]{27} = \sqrt[3]{3 \bullet 3 \bullet 3} = 3$$

a group of three 3's under a cube root reduce to the whole number 3

Example 3

$\sqrt[3]{64}$ asks "What number used as a factor 3 times is equal to 64?

$$\sqrt[3]{64} = \sqrt[3]{4 \bullet 4 \bullet 4} = 4$$

a group of three 4's under a cube root reduce to the whole number 4

The Cube Roots of Negative Numbers Exist

Example 4

$\sqrt[3]{-1}$ asks "What number used as a factor 3 times is equal to –1?

$$\sqrt[3]{-1} = \sqrt[3]{-1 \bullet -1 \bullet -1} = -1$$

a group of three -1's under a cube root reduce to the integer -1

Example 5

$\sqrt[3]{-8}$ asks "What number used as a factor 3 times is equal to –8?

$$\sqrt[3]{-8} = \sqrt[3]{-2 \bullet -2 \bullet -2} = -2$$

a group of three -2's under a cube root reduce to the integer -2

Example 6

$\sqrt[3]{-27}$ asks "What number used as a factor 3 times is equal to –27?

$$\sqrt[3]{-27} = \sqrt[3]{-3 \bullet -3 \bullet -3} = -3$$

a group of three -3's under a cube root reduce to the integer -3

$\sqrt[3]{-64}$ asks "What number used as a factor 3 times is equal to 64?

$$\sqrt[3]{-64} = \sqrt[3]{-4 \bullet -4 \bullet -4} = -4$$

a group of three -4's under a cube root reduce to the integer -4

$\sqrt[3]{-125}$ asks "What number used as a factor 3 times is equal to –125?

$$\sqrt[3]{-125} = \sqrt[3]{-5 \bullet -5 \bullet -5} = -5$$

a group of three -5's under a cube root reduce to the integer -5

The most common perfect cubes used it this class are shown below.

$\sqrt[3]{1} = 1$ $\sqrt[3]{8} = 2$ $\sqrt[3]{27} = 3$ $\sqrt[3]{64} = 4$ $\sqrt[3]{125} = 5$

$\sqrt[3]{-1} = -1$ $\sqrt[3]{-8} = -2$ $\sqrt[3]{-27} = -3$ $\sqrt[3]{-64} = -4$ $\sqrt[3]{-125} = -5$

Simplifying Cube Root Radicals using PERFECT CUBE FACTORS

If you can find the **largest** perfect cube factor of the radicand then reducing the radical expression is a short process. It requires that you find the largest perfect cube that is a factor of the original radicand.

Look for the largest perfect cube that is a factor of the radicand. Factor than reduce.

Example 7

Simplify: $\sqrt[3]{24}$

$= \sqrt[3]{8 \bullet 3}$
$= \sqrt[3]{8} \bullet \sqrt[3]{3}$
$= 2\sqrt[3]{3}$

Example 8

Simplify: $\sqrt[3]{81}$

$= \sqrt[3]{27 \bullet 3}$
$= \sqrt[3]{27} \bullet \sqrt[3]{3}$
$= 3\sqrt[3]{3}$

Example 9

Simplify: $\sqrt[3]{128}$

$= \sqrt[3]{64 \bullet 2}$
$= \sqrt[3]{64} \bullet \sqrt[3]{2}$
$= 4\sqrt[3]{2}$

Example 10

Simplify: $\sqrt[3]{-54}$

$= \sqrt[3]{-27 \bullet 2}$
$= \sqrt[3]{-27} \bullet \sqrt[3]{2}$
$= -3\sqrt[3]{2}$

Example 11

Simplify: $\sqrt[3]{-16}$

$= \sqrt[3]{-8 \bullet 2}$
$= \sqrt[3]{-8} \bullet \sqrt[3]{2}$
$= -2\sqrt[3]{2}$

Example 12

Simplify: $\sqrt[3]{-40}$

$= \sqrt[3]{-8 \bullet 5}$
$= \sqrt[3]{-8} \bullet \sqrt[3]{5}$
$= -2\sqrt[3]{5}$

The cube root of a negative number can always be reduced by using using $\sqrt[3]{-1} = -1$

Example 13

Simplify: $\sqrt[3]{-7}$

$= \sqrt[3]{-1 \bullet 7}$
$= \sqrt[3]{-1} \bullet \sqrt[3]{7}$
$= -\sqrt[3]{7}$

Example 14

Simplify: $\sqrt[3]{-9}$

$= \sqrt[3]{-1 \bullet 9}$
$= \sqrt[3]{-1} \bullet \sqrt[3]{9}$
$= -\sqrt[3]{9}$

Simplifying Cube Root Radicals using GROUPS OF 3 FACTORS

If you have a group of three factors under a cube root they can be reduced to a rational number.

Example 1

$\sqrt[3]{48}$

completly factor 48

$= \sqrt[3]{2 \bullet 2 \bullet 2 \bullet 2 \bullet 3}$

put each group of 3 factors under thier own cube root

$= \sqrt[3]{2 \bullet 2 \bullet 2} \bullet \sqrt[3]{2 \bullet 3}$

the group of 3 factors can be reduced

$= 2 \bullet \sqrt[3]{6}$

Example 2

$\sqrt[3]{104}$

completly factor 104

$= \sqrt[3]{3 \bullet 3 \bullet 3 \bullet 2 \bullet 2}$

put each group of 3 factors under thier own cube root

$= \sqrt[3]{2 \bullet 2 \bullet 2} \bullet \sqrt[3]{2 \bullet 2}$

the group of 3 factors can be reduced

$= 2 \bullet \sqrt[3]{4}$

Example 3

$\sqrt[3]{80}$

completly factor 80

$= \sqrt[3]{2 \bullet 2 \bullet 2 \bullet 2 \bullet 5}$

put each group of 3 factors under thier own cube root

$= \sqrt[3]{2 \bullet 2 \bullet 2} \bullet \sqrt[3]{2 \bullet 5}$

the group of 3 factors can be reduced

$= 2 \bullet \sqrt[3]{10}$

The cube root of a negative number can always be reduced by using using $\sqrt[3]{-1} = -1$

Example 4

$\sqrt[3]{-80}$

completly factor 104

$= \sqrt[3]{-1 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 5}$

put each group of 3 factors under thier own cube root

$= \sqrt[3]{-1} \bullet \sqrt[3]{2 \bullet 2 \bullet 2} \bullet \sqrt[3]{2 \bullet 5}$

the group of 3 factors can be reduced

$= -2 \bullet \sqrt[3]{10}$

Example 5

$\sqrt[3]{-54}$

completly factor 104

$= \sqrt[3]{-1 \bullet 3 \bullet 3 \bullet 3 \bullet 2}$

put each group of 3 factors under thier own cube root

$= \sqrt[3]{-1} \bullet \sqrt[3]{3 \bullet 3 \bullet 3} \bullet \sqrt[3]{2}$

the group of 3 factors can be reduced

$= -3 \bullet \sqrt[3]{2}$

Example 6

$\sqrt[3]{-32}$

completly factor -32

$= \sqrt[3]{-1 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2}$

put each group of 3 factors under thier own cube root

$= \sqrt[3]{-1} \bullet \sqrt[3]{2 \bullet 2 \bullet 2} \bullet \sqrt[3]{2 \bullet 2}$

the group of 3 factors can be reduced

$= -2 \bullet \sqrt[3]{4}$