Section 7 – 2: **Radical Expressions with Rational Exponents**

Change a Radical Expression into Exponential form

Change an expression with a radical sign into an expression in exponential form by writing the expression under the radical (radicand) to a fractional power that represents the index of the radical.

$$\sqrt[\text{index}]{\text{radicand}} = (\text{radicand})^{\text{index}}$$

$\sqrt{x} = x^{1/2}$

$\sqrt{x}$ means x raised to the $1/2$ power

$\sqrt[3]{5x} = (5x)^{1/3}$

$\sqrt[3]{5x}$ means 5x raised to the $1/3$ power

$\sqrt[4]{3y} = (3y)^{1/4}$

$\sqrt[4]{x}$ means 3y raised to the $1/4$ power

Example 1

$\sqrt{y} = y^{\frac{1}{2}}$

Example 2

$\sqrt{5x} = (5x)^{\frac{1}{2}}$

Example 3

$\sqrt[3]{4y} = (4y)^{\frac{1}{3}}$

$$\sqrt[\text{index}]{x^{\text{power}}} = x^{\frac{\text{power}}{\text{index}}}$$

Example 4

$\sqrt{x^8} = \sqrt[2]{x^8} = x^4$

$\sqrt[2]{x^8}$ means x^8 raised to the $1/2$ power

$\left(x^8\right)^{1/2} = x^{8\bullet\frac{1}{2}} = x^4$

Example 5

$\sqrt{x^3} = \sqrt[2]{x^3} = x^{3/2}$

$\sqrt[2]{x^3}$ means x^3 raised to the $1/2$ power

$\left(x^3\right)^{1/2} = x^{3\bullet\frac{1}{2}} = x^{\frac{3}{2}}$

Example 6

$\sqrt{x^5} = \sqrt[2]{x^5} = x^{5/2}$

$\sqrt[2]{x^5}$ means x^5 raised to the $1/2$ power

$\left(x^5\right)^{1/2} = x^{5\bullet\frac{1}{2}} = x^{\frac{5}{2}}$

Example 7

$\sqrt[3]{x^7} = x^{7/3}$

$\sqrt[3]{x^7}$ means x^7 raised to the $1/3$ power

$\left(x^7\right)^{1/3} = x^{7\bullet\frac{1}{3}} = x^{\frac{7}{3}}$

Example 8

$\sqrt[3]{x^6} = x^2$

$\sqrt[3]{x^6}$ means x^6 raised to the $1/3$ power

$\left(x^6\right)^{1/3} = x^{6\bullet\frac{1}{3}} = x^{\frac{6}{3}}$

Example 9

$\sqrt[4]{x^6} = x^{4/6} = x^{2/3}$

$\sqrt[4]{x^6}$ means x^6 raised to the $1/4$ power

$\left(x^6\right)^{1/4} = x^{6\bullet\frac{1}{4}} = x^{\frac{2}{3}}$

Change an Exponential Expression into Radical Form

$$x^{\frac{\text{power}}{\text{index}}} = \sqrt[\text{index}]{x^{\text{power}}}$$

Example 10

$x^{5/4} = \sqrt[4]{x^5} =$

Example 11

$y^{5/2} = \sqrt[2]{y^5} = \sqrt{y^5}$

Example 12

$(3x)^{2/3} = \sqrt[3]{(3x)^2}$

The Product Rule with Radical Expressions

$$x^A \bullet x^B = x^{A+B}$$

Change each expression with a radical sign into an expression in exponential form. Then use the product rule to combine the fractional exponents. Answer in both **exponential and radical form.**

Example 13

$\sqrt[3]{x} \bullet \sqrt{x^4}$

$= x^{\frac{1}{3}} \bullet x^{\frac{1}{4}}$

$= x^{\frac{1}{3}+\frac{1}{4}}$

$= x^{\frac{7}{12}}$

$= \sqrt[12]{x^7}$

Example 14

$\sqrt[4]{x^2} \bullet \sqrt[4]{x^3}$

$= x^{\frac{2}{4}} \bullet x^{\frac{3}{4}}$

$= x^{\frac{2}{4}+\frac{3}{4}}$

$= x^{\frac{5}{4}}$

$= \sqrt[4]{x^5}$

Example 15

$\sqrt[3]{x^2} \bullet \sqrt[3]{x^4}$

$= x^{\frac{2}{3}} \bullet x^{\frac{4}{3}}$

$= x^{\frac{2}{3}+\frac{4}{3}}$

$= x^{\frac{6}{3}}$

$= x^2$

The Quotient Rule for Mononominal terms

If the top exponent > bottom exponent

$$\frac{x^{A}}{x^{B}} = x^{A-B}$$

If the bottom exponent > top exponent

$$\frac{x^{A}}{x^{B}} = \frac{1}{x^{B-A}}$$

Change each expression with a radical sign into an expression in exponential form. Then use the quotient t rule to combine the fractional exponents. Answer in both **exponential and radical form.**

Example 16

$$\frac{\sqrt[3]{x}}{\sqrt[3]{x^{2}}}$$

$$= \frac{x^{\frac{1}{3}}}{x^{\frac{2}{3}}}$$

$$= \frac{1}{x^{\frac{2}{3}-\frac{1}{3}}}$$

$$= \frac{1}{x^{\frac{1}{3}}}$$

$$= \frac{1}{\sqrt[3]{x}}$$

Example 17

$$\frac{\sqrt[4]{x^{5}}}{\sqrt[4]{x^{3}}}$$

$$= \frac{x^{\frac{1}{4}}}{x^{\frac{5}{4}}}$$

$$= x^{\frac{5}{4}-\frac{3}{4}}$$

$$= x^{\frac{2}{4}}$$

$$= x^{\frac{1}{2}} = \sqrt{x}$$

Example 18

$$\frac{\sqrt[3]{x}}{\sqrt[6]{x^{5}}}$$

$$= \frac{x^{\frac{1}{3}}}{x^{\frac{5}{6}}}$$

$$= \frac{1}{x^{\frac{5}{6}-\frac{1}{3}}}$$

$$= \frac{1}{x^{\frac{3}{6}}}$$

$$= \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$$

