Section 7 – 1B: Radical Expressions with Fractional Exponents

The $\sqrt{}$ symbol is called a **radical sign**. Radical Expressions are written with a radical sign over an expression. The expression under the radical sign is called the Radicand. A number is placed outside and above the $\sqrt{}$ symbol and is called the index.

$$\sqrt[\text{idex}]{\text{radicand}}$$

The index states the type of root the radical sign represents

$\sqrt[2]{x}$ means take the **Square Root** of x

$\sqrt[2]{9} = 3$

$\sqrt[3]{x}$ means take the **Cube Root** of x

$\sqrt[3]{8} = 2$

$\sqrt[4]{x}$ means take the **Forth Root** of x

$\sqrt[4]{16} = 2$

$$\sqrt{x} = \sqrt[2]{x}$$

The expression $\sqrt{x}$ does not have an index shown. If no number is shown in the index we assume the index is a 2. $\sqrt{x} = \sqrt[2]{x}$ This is why many people call the $\sqrt{}$ symbol a Square Root Symbol. The $\sqrt{}$ symbol is only a square root symbol when it has a 2 for an index or no index is shown.

Finding the Square Root of a Perfect Square

$\sqrt{a}$ asks you "What positive number **times itself** is equal to a?

Example 1

Find $\sqrt{9}$

$9 = 3 \bullet 3$ so

$\sqrt{9} = 3$

Example 2

Find $\sqrt{\frac{25}{4}}$

$\frac{25}{4} = \frac{5}{2} \bullet \frac{5}{2}$ so

$\sqrt{\frac{25}{4}} = \frac{5}{2}$

Example 3

Find $\sqrt{\frac{81}{49}}$

$\frac{81}{49} = \frac{9}{7} \bullet \frac{9}{7}$ so

$\sqrt{\frac{81}{49}} = \frac{9}{7}$

Finding the Cube Root of a Perfect Cube

$\sqrt[3]{a}$ asks you "What positive number **as a factor 3 times** is equal to a?

Example 4

Find $\sqrt[3]{64}$

$64 = 4 \bullet 4 \bullet 4$ so

$\sqrt[3]{64} = 4$

Example 5

Find $\sqrt[3]{\frac{8}{27}}$

$\frac{8}{27} = \frac{2}{3} \bullet \frac{2}{3} \bullet \frac{2}{3}$ so

$\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

Example 6

Find $\sqrt[3]{\frac{64}{125}}$

$\frac{64}{125} = \frac{4}{5} \bullet \frac{4}{5} \bullet \frac{4}{5}$ so

$\sqrt[3]{\frac{64}{125}} = \frac{4}{5}$

Finding Fourth and Fifth Roots

$\sqrt[4]{a}$ asks you "What positive number **as a factor 4 times** is equal to a?

$\sqrt[5]{a}$ asks you "What positive number **as a factor 5 times** is equal to a?

Example 7

Find $\sqrt[4]{81}$

$81 = 3 \bullet 3 \bullet 3 \bullet 3$ so

$\sqrt[4]{81} = 3$

Example 8

Find $\sqrt[4]{16}$

$16 = 2 \bullet 2 \bullet 2 \bullet 2$ so

$\sqrt[4]{16} = 2$

Example 9

Find $\sqrt[5]{32}$

$32 = 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2$ so

$\sqrt[5]{32} = 2$

Fractional Exponents

There is a direct relationship between Square Roots, Cube Roots and Rational Exponents.

The denominator of a fractional exponent is the index of the radical

$x^{1/2} = \sqrt{x}$

$x^{1/2}$ means take the square root of x

$x^{1/3} = \sqrt[3]{x}$

$x^{1/3}$ means take the cube root of x

$x^{1/4} = \sqrt[4]{x}$

$x^{1/4}$ means take the fourth root of x

Example 1

Simplify $(9)^{1/2}$

$(9)^{1/2}$ means $\sqrt{9}$

take the square root of 9

$(9)^{1/2} = \sqrt{9} = 3$

Example 2

Simplify $(8)^{1/3}$

$(8)^{1/3}$ means $\sqrt[3]{8}$

take the cube root of 8

$(8)^{1/3} = \sqrt[3]{8} = 2$

Example 3

Simplify $(16)^{1/4}$

$(16)^{1/4}$ means $\sqrt[4]{16}$

take the fourth root of 16

$(16)^{1/4} = \sqrt[4]{16} = 2$

Example 4

Simplify $\left(\frac{25}{36}\right)^{1/2}$

$\left(\frac{25}{36}\right)^{1/2}$ means $\sqrt{\frac{25}{36}}$

take the square root of $\frac{25}{36}$

$\left(\frac{25}{36}\right)^{1/2} = \sqrt{\frac{25}{36}} = \frac{5}{6}$

Example 5

Simplify $\left(\frac{-8}{27}\right)^{1/3}$

$\left(\frac{-8}{27}\right)^{1/3}$ means $\sqrt[3]{\frac{-8}{27}}$

take the cube root of $\frac{-8}{27}$

$\left(\frac{-8}{27}\right)^{1/3} = \sqrt[3]{\frac{-8}{27}} = \frac{-2}{3}$

Example 6

Simplify $\left(\frac{16}{81}\right)^{1/4}$

$\left(\frac{16}{81}\right)^{1/4}$ means $\sqrt[4]{\frac{16}{81}}$

take the fourth root of $\frac{16}{81}$

$\left(\frac{16}{81}\right)^{1/4} = \sqrt[4]{\frac{16}{81}} = \frac{2}{3}$

The denominator of a fractional exponent is the index of the radical
Step 1: FInd the root of the base first.

The numarator of a fractional exponent power of the base
Step 2: FInd the power of the new base second.

Example 7

Simplify $9^{3/2}$

$9^{3/2} = \left(9^{1/2}\right)^3$

means take the
square root of 9
and then cube the answer

$9^{3/2} = \left(9^{1/2}\right)^3 = (3)^3$

$9^{3/2} = 27$

Example 8

Simplify $25^{3/2}$

$25^{3/2} = \left(25^{1/2}\right)^3$

means take the
square root of 25
and then cube the answer

$25^{3/2} = \left(25^{1/2}\right)^3 = (5)^3$

$25^{3/2} = 125$

Example 9

Simplify $27^{2/3}$

$27^{2/3} = \left(27^{1/3}\right)^2$

means take the
cube root of 27
and then square the answer

$27^{2/3} = \left(27^{1/3}\right)^2 = (3)^2$

$27^{2/3} = 9$

Example 10

Simplify $\left(\frac{4}{9}\right)^{3/2}$

means take the
square root of $4/9$
and then cube the answer

$\left(\frac{4}{9}\right)^{3/2} = \left(\sqrt{\frac{4}{9}}\right)^3 = \left(\frac{2}{3}\right)^3$

$\left(\frac{4}{9}\right)^{3/2} = \frac{8}{27}$

Example 11

Simplify $\left(\frac{8}{27}\right)^{2/3}$

means take the
cube root of $8/27$
and then square the answer

$\left(\frac{8}{27}\right)^{2/3} = \left(\sqrt[3]{\frac{8}{27}}\right)^2 = \left(\frac{2}{3}\right)^2$

$\left(\frac{8}{27}\right)^{2/3} = \frac{4}{9}$

Example 12

Simplify $\left(\frac{81}{16}\right)^{3/4}$

means take the
fourth root of $81/16$
and then cube the answer

$\left(\frac{81}{16}\right)^{3/4} = \left(\sqrt[4]{\frac{81}{16}}\right)^3 = \left(\frac{3}{2}\right)^3$

$\left(\frac{81}{16}\right)^{3/4} = \frac{27}{8}$

Negative Exponents Examples

Example 13

Simplify $\left(\frac{4}{9}\right)^{-3/2}$

means take the
square root of 4 / 9
and then cube the answer
and then "flip" the answer

$$\left(\frac{4}{9}\right)^{-3/2} = \left(\sqrt{\frac{4}{9}}\right)^{-3} = \left(\frac{2}{3}\right)^{-3}$$

$$\left(\frac{2}{3}\right)^{-3} = \left(\left(\frac{2}{3}\right)^{3}\right)^{-1} = \left(\frac{8}{27}\right)^{-1}$$

$$\left(\frac{4}{9}\right)^{-3/2} = \frac{27}{8}$$

Example 14

Simplify $\left(\frac{64}{27}\right)^{-2/3}$

means take the
cube root of 64 / 27
and then square the answer
and then "flip" the answer

$$\left(\frac{64}{27}\right)^{-3/2} = \left(\sqrt[3]{\frac{64}{27}}\right)^{-2} = \left(\frac{4}{3}\right)^{-2}$$

$$\left(\frac{4}{3}\right)^{-2} = \left(\left(\frac{4}{3}\right)^{2}\right)^{-1} = \left(\frac{16}{27}\right)^{-1}$$

$$\left(\frac{64}{27}\right)^{-2/3} = \frac{27}{16}$$

