Section 7 – 0: **Exponents Review**

We have seen how an exponent can be used to shorten the way we write repeated products

x^3 means $(x)(x)(x)$ x^4 means $(x)(x)(x)(x)$ $3x^2$ means $3(x)(x)$

There are rules that can simplify our work when we **multiply several terms** with exponents.

The Product Rule for Mononominal Terms

$$\left(\mathrm{A}x^{\mathrm{C}}y^{\mathrm{D}}\right)\left(\mathrm{B}x^{\mathrm{E}}y^{\mathrm{F}}\right)=\mathrm{A}\bullet\mathrm{B}\bullet x^{\mathrm{C+E}}\bullet y^{\mathrm{D+F}}$$

To find the product of 2 **Mononominal terms**:

1. **Multiply the coefficients** of each term and use that product as the new coefficient.
2. To find the final exponent of each different **variable base** keep the **common base** and **add the exponents of each common base**. List the variable bases in alphabetical order.

Example 1

$\left(2\mathrm{y}^5\right)\left(3\mathrm{y}^6\right)$

$=2\cdot 3\mathrm{y}^{5+6}$

$=6\mathrm{y}^{11}$

Example 2

$\left(-x^2y^3\right)\left(xy^5\right)$

$=-1\bullet x^{2+1}y^{3+5}$

$=-x^3y^8$

Example 3

$\left(-2\mathrm{x}^4\mathrm{y}\right)\left(5\mathrm{xy}^5\right)$

$=-2\cdot 5\mathrm{x}^{4+1}\mathrm{y}^{1+5}$

$=-10\mathrm{x}^5\mathrm{y}^6$

The Power Rule for Mononominal terms

$$\left(\mathrm{A}^1x^{\mathrm{B}}y^{\mathrm{C}}\right)^{\mathrm{D}}=\mathrm{A}^{1\bullet\mathrm{D}}x^{\mathrm{B}\bullet\mathrm{D}}y^{\mathrm{C}\bullet\mathrm{D}}$$

Simplify a **Mononominal term** inside a parentheses that has been raised to an **exponent**.

1. If a constant or variable inside does not have an exponent put a 1 above it.
2. Multiply the exponent **outside** the parentheses by each exponent **inside** the parentheses
3. Simplify a constant that is raised to an exponent. $3^2=9$

Example 3

$\left(xy^5\right)^3$

$=x^{1\cdot 3}y^{5\cdot 3}$

$=x^3y^{15}$

Example 4

$\left(5^1\mathrm{x}^3\mathrm{y}^4\right)^2$

$=5^{1\cdot 2}\mathrm{x}^{3\cdot 2}\mathrm{y}^{4\cdot 2}$

$=25\mathrm{x}^6\mathrm{y}^8$

Example 5

$\left(2^1xy^5\right)^3$

$=2^3x^{1\cdot 3}\,3y^{5\cdot 3}$

$=8x^3y^{15}$

The Quotient Rule for Mononominal terms

If $T > B$

top exponent > bottom exponent

$$\frac{x^{T}}{x^{B}} = \frac{x^{T-B}}{1}$$

If $B > T$

bottom exponent > top exponent

$$\frac{x^{T}}{x^{B}} = \frac{1}{x^{B-T}}$$

1. Reduce the coefficients by canceling (reducing) the numbers just like a normal fraction
2. **Subtract** the **exponents** of each **common variable** and put the answer where the **largest exponent** for that variable **was**.
3. If the top and bottom exponent of a common variable are equal to each other then the variable cancels to a 1: Example $\frac{x^2}{x^2} = \frac{\cancel{x} \bullet \cancel{x}}{\cancel{x} \bullet \cancel{x}} = 1$

Example 7

$$\frac{15x^3y^2}{6xy^7}$$

$$= \frac{\cancel{15}^{5}}{\cancel{6}^{2}} \frac{x^{3-1}}{y^{7-2}}$$

$$= \frac{5x^2}{2y^5}$$

Example 8

$$\frac{15x^3y^8}{10x^6y^2}$$

$$= \frac{\cancel{15}^{3}}{\cancel{10}^{2}} \frac{y^{8-2}}{x^{6-3}}$$

$$= \frac{3y^6}{2x^3}$$

Example 9

$$\frac{3x^3y^2}{6x^8y^6}$$

$$= \frac{\cancel{3}^{1}}{\cancel{6}^{2}} \frac{1}{x^{8-3}y^{6-2}}$$

$$= \frac{1}{2x^5y^4}$$

Example 10

$$\frac{6x^6y^8}{2x^6y^4}$$

$$= \frac{\cancel{6}^{3}}{\cancel{3}^{1}} \frac{x^{6-6}y^{8-4}}{}$$

$$= \frac{3y^4}{1} = 3y^4$$

The Power Rule, Product Rule and Quotient Rules Together

The order of operations **PEMDAS** requires that we perform what's inside the parenthesis first, then perform any exponents and then perform multiplication. A fraction bar is considered a parenthesis with the top of the fraction inside one parenthesis and the bottom of the fraction inside another parenthesis. If the Power Rule and the Product Rule and Quotient Rule are in the same problem we perform the **Power Rule first** and then perform the **Product Rule second** on the numerator and then again on the denominator. Finish by performing the **Quotient Rule last.** Not every problem will have all three rules in it. If only two of the three rules are present then perform the ones present in the order listed below.

1. Power Rule First 2. Product Rule Second 3. Quotient Rule Last

Example 1

Power Rule

$$\left(3xy^2\right)^2\left(x^2y\right)^3$$

Product Rule

$$=\left(9x^2y^4\right)\left(x^6y^3\right)$$

$$=9x^8y^7$$

Example 2

Power Rule

$$\left(3x^2y^3\right)^2\left(2x^2y\right)$$

Product Rule

$$=\left(9x^4y^6\right)\left(2x^2y\right)$$

$$=18x^6y^7$$

Example 3

Power Rule

$$\left(2xy^2\right)^3\left(3x^2y^3\right)^2$$

Product Rule

$$=\left(8x^3y^6\right)\left(9x^2y^3\right)$$

$$=72x^5y^9$$

Example 4

Power Rule

$$\frac{\left(4x^3\right)^2}{\left(2x^5\right)^3}$$

Quotent Rule

$$=\frac{16x^6}{8x^{15}}$$

$$=\frac{2}{x^9}$$

Example 5

Product Rule

$$\frac{2x^{10}}{\left(3x^5\right)\left(4x^3\right)}$$

Quotent Rule

$$=\frac{2x^{10}}{12x^8}$$

$$=\frac{x^2}{6}$$

Example 6

Power Rule

$$\frac{\left(5x^4\right)^2}{\left(2x^2\right)^3}$$

Quotent Rule

$$=\frac{25x^8}{8x^6}$$

$$=\frac{25x^2}{8}$$

Negative Exponents Rule for Mononominal terms

Changing a Negative Exponent into Positive Exponent.

If x is any non zero number then $x^{-1} = \frac{1}{x}$ and $\frac{1}{x^{-1}} = x$

If abase with a **negative exponent** is in the numerator of a fraction it must be moved to the denominator and the exponent made positive. If a base with a **negative exponent** is in the denominator of a fraction it must be moved to the numerator and the exponent made positive.

Example 1

$$x^{-2} = \frac{1}{x^2}$$

x^{-2} was moved to the bottom

Example 2

$$\frac{1}{x^{-4}} = x^4$$

x^{-4} was moved to the top

Example 3

$$\frac{y^{-3}}{x^{-2}} = \frac{x^2}{y^3}$$

y^{-3} was moved to the bottom

x^{-2} was moved to the top

A Constant with a Negative Exponent

A Constant with negative **exponent** is treated just like a variable with a negative exponent.

A constant with a negative number in front **DOES NOT MOVE.**

Example 6

$$\frac{1}{2^{-3}} = 2^3 = 8$$

2^{-3} was moved to the top

Example

$$\frac{3^{-2}}{4} = \frac{1}{4 \bullet 3^2} = \frac{1}{36}$$

3^{-2} was moved to the bottom

Example 8

$$\frac{2^{-3}}{5^{-2}} = \frac{5^2}{2^3} = \frac{25}{8}$$

2^{-3} was moved to the bottom

5^{-2} was moved to the top

Example 9

$$\frac{-3}{4x^{-2}} = \frac{-3x^2}{4}$$

the -3 is NOT moved to the bottom
because it does NOT have a negative exponent

the x^{-2} IS moved to the top
becaquse it does have a negative exponent

Power Rule, Product Rule, Negative Exponents, Quotient Rule

Putting them all together

The order of operations **PEMDAS** requires that we **perform the Power Rule first.** After the power rule has been performed there are several options for the order of the remaining rules to be performed. A common order is listed below.

Step 1: **The Power Rule must be done first.** This is a requirement of PEMDAS

Step 2: Perform the **Product Rule** if there is a product of common variables.

Step 3: Move a base with a **Negative Exponent.**

Step 4: Perform **the Quotient Rule** If there is a common variable on the **top and bottom** of the fraction.

Some students will move a base with a negative exponent second. They will then use what ever combination of product and quotient rules are needed to finish the process. Either of the suggested orders will work but the power Rule must always be performed first.

Example 1

Step 1. Negative Exponents

Step 2. Quotient Rule

$$\frac{x^{-7}y^{-4}}{x^{-4}y^{-9}}$$

move the bases with negitive exponents

$$=\frac{x^4y^9}{x^7y^4}$$

use the quotient rule for

$$\frac{x^4}{x^7} \text{ and } \frac{y^9}{y^4}$$

$$=\frac{y^5}{x^3}$$

Example 2

Step 1. Negative Exponents

Step 2. Quotient Rule

$$\frac{x^5y^{-1}}{3^{-2}x^6y^{-7}}$$

move the bases with negitive exponents

$$=\frac{3^2x^5y^7}{x^6y}$$

use the quotient rule for

$$\frac{x^5}{x^6} \text{ and } \frac{y^7}{y}$$

$$=\frac{9y^6}{x}$$

Example 3

1. Power Rule
2. Negative Exponents

$$\left(2xy^{-2}\right)^{-3}$$

perform the Power Rule by multipling each exponent inside by the exponent outside

$$\left(2^1x^2y^{-2}\right)^{-3}$$

$$=2^{-3}x^{-3}y^6$$

move the bases with negitive exponents

$$=\frac{y^6}{8x^3}$$

Example 4

1. Power Rule
2. Negative Exponents

$$\frac{1}{\left(3x^2y^{-3}\right)^{-2}}$$

perform the Power Rule by multipling each exponent inside by the exponent outside

$$\frac{1}{\left(3^1x^2y^{-3}\right)^{-2}}$$

$$\frac{1}{3^{-2}x^{-4}y^6}$$

move the bases with negitive exponents

$$=\frac{9x^4}{y^6}$$

Example 5

1. Product Rule
2. Negative Exponents

$$\left(5x^{-2}y^5\right)\left(-3x^{-5}y^{-2}\right)$$

perform the Product Rule

$$-15x^{-2-5}y^{5-2}$$

$$=-15x^{-7}y^3$$

move the bases with negitive exponents

$$=\frac{-15y^3}{x^7}$$

Example 6

1. Product Rule
2. Negative Exponents

$$\frac{1}{\left(2x^{-3}y^{-6}\right)\left(4x^5y^{-3}\right)}$$

perform the Product Rule

$$\frac{1}{8x^{-3+5}y^{-6-3}}$$

$$\frac{1}{8x^2y^{-9}}$$

move the bases with negitive exponents

$$=\frac{y^9}{8x^2}$$

Example 7

1. Product Rule
2. Negative Exponents
3. Quotient Rule and Product Rule

$$\frac{\left(x^{-3}y^{-5}\right)\left(x^{-2}y\right)}{x^{3}y^{-7}}$$

$$=\frac{x^{-5}y^{-4}}{x^{3}y^{-7}}$$

$$=\frac{y^{7}}{x^{5}x^{3}y^{4}}$$

$$=\frac{y^{3}}{x^{8}}$$

Example 8

1. Product Rule
2. Negative Exponents
3. Quotient Rule and Product Rule

$$\frac{2x^{4}y^{-2}}{\left(3x^{8}y^{-2}\right)\left(4x^{-2}y^{-3}\right)}$$

$$=\frac{2x^{4}y^{-2}}{12x^{6}y^{-5}}$$

$$=\frac{2x^{4}y^{5}}{12x^{6}y^{2}}$$

$$=\frac{y^{3}}{6x^{2}}$$

Some students will move a base with a negative exponent second. They will then use what ever combination of product and quotient rules are needed to finish the process. Either of the suggested orders will work but the power Rule must always be performed first.

Example 9

1. Power rule
2. Negative Exponents
3. Product Rule Product Rule

$$\frac{\left(x^{2}y^{-3}\right)^{-2}}{\left(xy^{-3}\right)^{3}}$$

$$=\frac{x^{-4}y^{6}}{x^{3}y^{-9}}$$

$$=\frac{y^{6}y^{9}}{x^{4}x^{3}}$$

$$=\frac{y^{15}}{x^{7}}$$

Example 10

1. Power Rule
2. Negative Exponents
3. Quotient Rule and Product Rule

$$\frac{\left(3x^{-4}y^{-1}\right)^{2}}{\left(x^{2}y^{-3}\right)^{-3}}$$

$$=\frac{9x^{-8}y^{-2}}{x^{-6}y^{9}}$$

$$=\frac{9x^{6}}{x^{8}y^{2}y^{9}}$$

$$=\frac{9}{x^{2}y^{11}}$$

