A Function is a relation that requires that

\textbf{for every x value}

\textbf{there is only one value of y related to that x}

The definition of a function requires that each x value has only one y value related to it. This is important in many algebraic operations. If you put $x = 2$ into the equation $y = x + 1$ then you would expect to get a y value of 8. This means that 2 is related to 3 by the function $y = x + 1$. Each time you put 2 in for x you would expect an outcome of 3.

Set $A = \{(2, 3), (4, 5), (6, 7)\}$

Set $A$ is a function because every x value is related to only one y value.

**A function can have more than one x value related to the same y value**

Set $B = \{(2, 4), (2, -4), (3, 9), (3, 9)\}$

Set $B$ is a function because every x value is related to only one y value. The y values are related to more than one x value but that is allowed for functions.

If you think of the x values as boys and the y values as girls then a function can be thought of as a rule that says

any one boy can only date one girl
but a girl can date more than one boy.
One To One Functions

Set $C = \{ (4, 2) (5, 3) (6, 7) (8, 9) \}$

Set $C$ is a function because every $x$ value is related to only one $y$ value

But Set $C$ also has every $y$ value related to only one $x$ value

Functions where every $x$ has only one $y$ AND every $y$ has only one $x$ value are **special functions**. These functions are called One to One Functions

One to One Functions

A One to One Function has each $x$ value related to a single $y$ value and that $y$ value is related to only that $x$ value.

Each $x$ value is related to one $y$ value each other and to no other values.

If you think of the $x$ values as boys and the $y$ values as girls then a One To One Function can be thought of as a rule that says any one boy can only date one girl AND that girl can only date that boy.

A One to One Function requires **total monogamy between $x$ and $y$ values**.
Determine if a Relation is a One To One Function

Step One: Test to see if the Relation is a Function
Test to see that for every x value there is only one value of y related to that x.

Step Two: Test to see if the Function from step 1 is One to One.
Test to see that each x value is related to a single y value and that y value is related to only that x value.

Example 1
Set A:  (2, 3) (9, 2) (2, 8) (0, 6)
Set A is Not a Function
x = 2 is related to both
y = 3 and y = 8 values
Fails Step 1
Set A is NOT One to One

Example 2
Set B:  (7, 4) (2, 3) (1, 1) (3, -3)
Set B is a One to One Function
each x value is related to a single y value and that y value is related to only that x value.

Example 3
Set C:  (2, 3) (9, 2) (2, 6) (5, 6)
Set C is Not One to One
y = 6 is related to both
both x = 2 and x = 5
Fails Step 2

Example 4
Set D:  (1, 4) (2, 3) (5, 7) (7, 5)
Set D is a One to One Function
each x value is related to a single y value and that y value is related to only that x value.

Determine if a Table of Ordered pairs is A One To One Function

Example 5
Set E.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>-4</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>8</td>
<td>-6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

Set E is Not One to One
y = 8 is related to both
both x = 2 and x = 9
Fails Step 2

Example 6
Set F.

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>-2</th>
<th>-4</th>
<th>6</th>
<th>-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-2</td>
<td>3</td>
<td>1</td>
<td>8</td>
<td>-5</td>
</tr>
</tbody>
</table>

Set F is a One to One Function
each x value is related to a single y value and that y value is related to only that x value.
Determine if a Mapping of Ordered pairs is a One To One Function

Example 7
Set G

Set G is Not a Function
x = 2 is related to both
y = 3 and y = 8 values
Fails Step 1
Set G is NOT One to One

Example 8
Set H

Set H is a One to One Function
each x value is related to a single y value
and that y value is related to only that x value.

Determine if a Graph Represents a One To One Function

Use the Vertical Test to determine if a graph represents a function

Step 1: If a vertical line hits the graph at more than one point then the graph does Not Represent A Function

Step 2: After a graph has passed the Vertical Line test you can then use a Horizontal Line Test to determine if the function is One to One

If a Horizontal line hits the graph of a Function at more than one point then the graph does Not Represent A One To One Function
Example 1

First use the vertical line test to see if the graph is a function.

This graph passes the Vertical Line Test so it is a Function.

Then use the Horizontal Line Test to see if the function is One To One.

This graph passes the Horizontal Line Test so the Function is a One To One Function.
Example 2

First use the vertical line test to see if the graph is a function. If a vertical line intersects the graph at more than one point, the graph does not represent a function.

This graph passes the Vertical Line Test so it IS a Function.

Then use the Horizontal Line Test to see if the function is One To One. If a horizontal line intersects the graph at more than one point, the function is not One To One.

This graph fails the Horizontal Line Test so it IS NOT a One To One Function.
Example 3

First use the vertical line test to see if the graph is a function

This graph passes the Vertical Line Test so it **IS a Function**

then use the Horizontal Line Test to see if the function is One To One

This graph **fails** the Horizontal Line Test so it **IS NOT a One To One Function**
Example 4

First use the vertical line test to see if the graph is a function

then use the Horizontal Line Test to see if the function is One To One

This graph **Fails** the Vertical Line Test so it **IS NOT a Function**

If a graph does not represent a function then it **cannot be a**
One to One Function