Relations

A Relation is any set of ordered pairs

We use a capital letter to name a set and put the ordered pairs of the set inside brackets like \{ \}

Domain and Range

The first elements of each ordered pair will be called the Domain
we will let the values for the domain be represented by the variable \( x \)

The second elements each ordered pair will be called the Range
we will let the values for the range be represented by the variable \( y \)

\[
Set \ A = \{ ( -2 , -1 ) \hspace{1em} ( 1 , 2 ) \hspace{1em} ( 3 , 4 ) \} \\
\]

Set \( A \) contains 3 ordered pairs.

The ordered pair (\( -2 , -1 \)) means \(-2\) is related to \(-1\)

The ordered pair (\( 1 , 2 \)) means \(1\) is related to \(2\)

The ordered pair (\( 3 , 4 \)) means \(3\) is related to \(4\)

The Domain is \( x = -2 , 1 \) and \( 3 \)

The Range is \( y = -1 , 2 \) and \( 4 \)

There are several other formats that this relation can be expressed in. Each of the different formats still expresses the same relation as above but in a different way.

A Relation is any set of ordered pairs, any table of ordered pairs, any mapping of ordered pairs, any equation that relates ordered pairs or any graph of ordered pairs.

A Relation is any table of ordered pairs.

\[
\begin{array}{c|c|c}
\hline
x & -2 & 1 & 3 \\
\hline
y & -1 & 2 & 4 \\
\hline
\end{array}
\]

The \( x, y \) table means \(-2\) is related to \(-1\) \((-2, \ -1)\)

The \( x, y \) table means \(1\) is related to \(2\) \((1, \ 2)\)

The \( x, y \) table means \(3\) is related to \(4\) \((3, \ 4)\)

The Domain is \( x = -2 , 1 \) and \( 3 \)

The Range is \( y = -1 , 2 \) and \( 4 \)
A Relation is any **mapping** of ordered pairs

\[
\begin{array}{c|c}
\text{x} & \text{y} \\
-2 & -1 \\
1 & 2 \\
3 & 4 \\
\end{array}
\]

The mapping line from \(-2\) to \(-1\) means \(-2\) is related to \(-1\) \((-2, -1)\)

The mapping line from \(1\) to \(2\) means \(1\) is related to \(2\) \((1, 2)\)

The mapping line from \(3\) to \(4\) means \(3\) is related to \(4\) \((3, 4)\)

The **Domain** is \(x = -2, 1,\) and \(3\)

The **Range** is \(y = -1, 2,\) and \(4\)

A Relation is any **equation** that relates ordered pairs

\[y = x + 1\] for the \(x\) values of \(-2, 1,\) and \(3\)

The equation shows that if \(x = -2\) then \(y = -1\) \((-2, -1)\)

The equation shows that if \(x = 1\) then \(y = 2\) \((1, 2)\)

The equation shows that if \(x = 3\) then \(y = 4\) \((3, 4)\)

The **Domain** is \(x = -2, 1,\) and \(3\)

The **Range** is \(y = -1, 2,\) and \(4\)

A Relation is any **graph** of ordered pairs.
This graph is the set of 3 **discrete** ordered pairs.

The graph is the ordered pairs \((-2,-1)\) \((1, 2)\) and \((3, 4)\)

The **Domain** is \(x = -2, 1,\) and \(3\)

The **Range** is \(y = -1, 2,\) and \(4\)
Functions

A Function is a Relation that requires
that every x value has only one y value related to that x

If you put \( x = 5 \) into the equation \( y = x + 3 \) you would expect to get a y value of 8 every time. The ordered pair \((5, 8)\) shows that an \( x \) value of 5 results in a y value of 8. If an ordered pair with an \( x \) value of 5 had a y value different than 8, like \((5, 10)\), that would mean an \( x \) value of 5 resulted in y values of both an 8 and a 10. If this happens then the relation is NOT a function. The definition of a function requires that each \( x \) value have only one \( y \) value related to it. This is important to many algebraic operations.

Not every set of ordered pairs is a function

If you think of the \( x \) values as boys and the \( y \) values as girls then a function can be thought of as a rule that says
any one boy can only date one girl
but the girls can date more than one boy.

Some Sets of Ordered Pairs Represent a Function

Example 1
Set A = \{ (5, 6) (7, 3) (4, 8) (1, –3) \}
Set A is a function because every single \( x \) value is related to only one \( y \) value.

Example 2
Set B = \{ (4, 2) (7, 3) (2, 6) (4, 8) \}
Set B is NOT a function because and \( x \) = 4 is related to both \( y \) = 2 and \( y \) = 8

Example 3
Set C
\[
\begin{array}{cccccc}
 x & 2 & -2 & -4 & 6 & -5 \\
 y & -2 & 3 & -2 & -2 & -5 \\
\end{array}
\]
Set C is a function because every single \( x \) value is related to one only one \( y \) value.

Example 4
Set D
\[
\begin{array}{cccccc}
 x & 1 & 3 & -4 & 6 & 1 \\
 y & -3 & 4 & -6 & 7 & 2 \\
\end{array}
\]
Set D is not a function because \( x \) = 1 is related to both \( y \) = –3 and \( y \) = 2
Some Mappings Represent a Function

**Example 1**

Mapping A is a function because every single x value is related to only one y value.

**Example 2**

Mapping B is NOT a function because x = 4 is related to both y = –2 and y = 2.

**Example 3**

Mapping C is NOT a function because x = 3 is related to both y = –2 and y = 5.

**Example 4**

Mapping D is a function because every single x value is related to only one y value.
Some Graphs Represent a Function

It is not easy to tell if an equation in two variables is a function. If the domain contained an infinite number of values for \( x \) then you would need to check to see if every value for \( x \) had only one \( y \) as an outcome. This is not practical. The best way to test to determine if an equation is a function is to know the shape of the graph. A simple test can then determine if the graph of the equation is a function or not.

The Vertical Line Test is used to determine if a graph represents a function.

If any single vertical line hits the graph at more than one point then the graph does Not Represent A Function.

Example 1
Is a Function because every vertical line hits the graph at only one point

![Example 1 Graph](image1)

Example 2
Is Not a Function because the vertical line hits the graph at more than one point

![Example 2 Graph](image2)

Example 3
Is Not a Function because the vertical line hits the graph at more than one point

![Example 3 Graph](image3)

Example 4
Is a Function because every vertical line hits the graph at only one point

![Example 4 Graph](image4)

Example 5
Is Not a Function because the vertical line hits the graph at more than one point

![Example 5 Graph](image5)

Example 6
Is a Function because the vertical line hits the graph at only one point

![Example 6 Graph](image6)
Some Equations Represent a Function

The **best way** to test to determine if an equation is a function is to know the shape of the graph. If you do not know the shape of a graph then it is very difficult to know if an equation represents a function. There are some cases where an equation never represent a function.

A. If the expression $|y|$ is in the equation then it is **Not a Function**.

B. If the expression $y^{\text{even power}}$ ($y^2, y^4, y^6 ...$) is in the equation then it is **Not a Function**.

C. If $x$ and $y$ are related by an inequality then the inequality is **Not a Function**.

D. Any vertical line ($x = 4, x = -5, x = 8$ etc.) is **Not a Function**.

The following equations are **Not a Function**.

**Example 1**

$y < x + 2$

is **Not a Function**

because it is an inequality

**Example 2**

$x = 2y^2 - 3$

is **Not a Function**

because it has $y^{\text{even power}}$

**Example 3**

$x^2 + 2y^6 = 18$

is **Not a Function**

because it has $y^{\text{even power}}$

**Example 4**

$x = 2|y|$

is **Not a Function**

because it has $|y|$

**Example 5**

$x = 5$

is **Not a Function**

because it is a **vertical line**

**Example 6**

$y \geq 2x^2 + 5$

is **Not a Function**

because it is an inequality