Section 5 – 5: Applications and Formulas

Work Rate Problems

A common algebra problem involves the time that it takes workers who work together to complete a job. This type of problem is called a **Work Rate Problem**. These problems involve two or more people who work together to complete one job. The most common form of the problem is to be given the time it would take each worker to complete the whole job if they worked alone and then find the time required to complete the whole job of they both worked together.

How do I find the how much of the job is completed in one hour for a worker?

If one person can complete a whole job in 6 hours then that person completes $\frac{1}{6}$ **of the job** per hour.

This rate can be written as $\frac{1}{6}\ \frac{\text{job}}{\text{hour}}$

If we multiply the rate by the hours the person works we get the **fraction of the job** that the person completes.

If the person above works for 3 hours they complete $\frac{1}{6}\ \frac{\text{job}}{\text{hour}} \bullet 3\ \text{hours}$ or $\frac{1}{2}$ the job

If the person above works for 5 hours they complete $\frac{1}{6}\ \frac{\text{job}}{\text{hour}} \bullet 5\ \text{hours}$ or $\frac{5}{6}$ the job

Simple **Work Rate Problems**

In a simple **Work Rate Problem** two (or more people) work for the same amount of time but at different rate. They each complete a fraction of the whole job. Together they complete the entire job.

You are given the time 2 different workers need to do the job if they each work alone.

Worker A can do the job in **x hours so his rate is** $\frac{1}{x}\ \frac{\text{job}}{\text{hour}}$

Worker B can do the job in **Y hours** $\frac{1}{y}\ \frac{\text{job}}{\text{hour}}$

If the time they work together is **t** then worker A completes $\frac{1}{x} \bullet t$ part of the job.

If the time they work together is **t** then worker B completes $\frac{1}{y} \bullet t$ part of the job.

The parts of the job that Worker A and Worker B complete must add up to 1 complete job

$$\frac{1}{x} \bullet t + \frac{1}{y} \bullet t = 1$$

where t is the time that they work together to complete the job

Example 1

Worker A can paint the wall in 3 hours alone. Worker B can paint the same wall in 4 hours alone. How long in minutes (find t) will it take to complete the job if they both work together?

Let T = hours that they work together

$$\frac{1}{3} \bullet t + \frac{1}{4} \bullet t = 1$$

Multiply all terms by 12

$$12 \bullet \frac{1}{3} \bullet t + 12 \bullet \frac{1}{4} \bullet t = 1 \bullet 12$$

$$4t + 3t = 12$$

$$7t = 12$$

$$t = \frac{7}{12} \text{ Hours}$$

(note 1/12 of an hour is 5 minutes) so

35 minutes

or convert $\frac{7}{12}$ Hours by unit conversion

$$\frac{7}{\cancel{12}} \cancel{\text{hour}} \bullet \frac{\cancel{60}^{5} \text{ min}}{1 \cancel{\text{hour}}} = 35 \text{minutes}$$

Example 2

Worker A can paint the wall in 10 hours alone. Worker B can paint the same wall in 8 hours alone. How long (in hours) will it take to complete the job if they both work together?

Let T = hours that they work together

$$\frac{1}{10} \bullet t + \frac{1}{8} \bullet t = 1$$

Multiply all terms by 40

$$40 \bullet \frac{1}{10} \bullet t + 40 \frac{1}{8} \bullet t = 1 \bullet 40$$

$$4t + 5t = 40$$

$$9t = 40$$

$$t = \frac{40}{9} \text{ Hours}$$

$$\text{or } 4\frac{4}{9} \text{ Hours}$$

Example 3

Worker A can paint the wall in 6 hours alone. Worker B do the same the same twice as fast. How long (find t) will it take if they both work together?

Worker A takes 6 hours
Worker B is twice as fast so he takes half as long or 3 hours.

$$\frac{1}{6} \bullet t + \frac{1}{3} \bullet t = 1$$

Multiply all terms by 6

$$6 \bullet \frac{1}{6} \bullet t + 6 \frac{1}{3} \bullet t = 1 \bullet 6$$

$$1t + 2t = 6$$

$$3t = 6$$

$$t = 2 \text{ Hours}$$

Example 4

Pump A can fill a tank in 6 hours. Drain B can empty the tank in 12 hours. How long will it take to fill the tank if the pump is working but the drain is open?

$$\frac{1}{6}\bullet t-\frac{1}{8}\bullet t=1$$

Multiply all terms by 24

$$24\bullet\frac{1}{6}\bullet t-24\bullet\frac{1}{8}\bullet t=1\bullet 24$$

$$4t-3t=24$$

$$t=24 \text{ Hours}$$

Resistors Example 1

When two resistors R_1 and R_2 are wired in parallel the total resistance R is $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$

If a 9 ohm resistor and a 6 ohm resistor are wired in parallel what is the **total resistance**?

R_1 **= 9 ohms**

R_2 **= 6 ohms**

$$\frac{1}{R}=\frac{1}{9}+\frac{1}{6}$$

multiply each term by $10R_2$

$$18R\bullet\frac{1}{R}=18R\bullet\frac{1}{9}+18R\bullet\frac{1}{6}$$

$$18=2R+3R$$

$$18=5R$$

$$R=\frac{18}{5}\text{ ohms}$$

Resistors Example 2

When two resistors R_1 and R_2 are wired in parallel the total resistance R is $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ If the total resistance is 2 ohms and one of the resistors is 5 ohms what is the other resistance?

R = 2 ohms

R_1 **= 5 ohms**

$$\frac{1}{2} = \frac{1}{5} + \frac{1}{R_2}$$

multiply each term by $10R_2$

$$10R_2 \bullet \frac{1}{2} = 10R_2 \bullet \frac{1}{5} + 10R_2 \bullet \frac{1}{R_2}$$

$$5R_2 = 2R_2 + 10$$

$$3R_2 = 10$$

$$R_2 = \frac{10}{3} \text{ ohms}$$

