Section 5 – 4: Solving Rational Equations

Solving Equations with Fractions

In section 2–3 we solved equations that had fractions with whole numbers for the denominators. In that section we solved the equations by first using the multiplication property of equality to multiply **every term in an equation** by the Least Common Denominator. When we multiply each term by the LCD the resulting equation will not have any fractions in it. We can then solve this equation by the methods of chapter 2.

Example

$\frac{4x}{5}+\frac{3}{2}=\frac{3x}{10}$ multiply all 3 terms by 10

$\left(\frac{10}{1}\right)\frac{4x}{5}+\left(\frac{10}{1}\right)\frac{3}{2}=\left(\frac{10}{1}\right)\frac{3x}{10}$

$8x+15=3x$ subtract 3x from both sides

$-3x \quad\quad -3x$

$5x+15=0$ subtract 15 from both sides

$\frac{5x}{5}=\frac{-15}{5}$ (divide both sides by 5)

$\mathbf{x=-3}$

Solving Rational Equations

All of the **Rational Equations** in this section will contain terms with denominators that contain polynomials instead of whole numbers.

$$\frac{1}{6}-\frac{x-5}{3x}=\frac{x+5}{4x} \qquad\qquad \frac{1}{x(x-1)}-\frac{x-5}{x}=\frac{x+5}{x-1}$$

When we solve the rational equations in this section we will use the same method as before. We start by multiplying **every term in an equation** by the Least Common Denominator. The difference in this section is that the LCD will not be a whole number as it was in section 2–3.

$$\frac{x-4}{2(x+2)}=\frac{5}{4} \qquad\qquad \frac{2}{3x}=\frac{x}{6}+\frac{-4}{(x+4)}$$

.

The LCD is $4(x+2)$ The LCD is $6x(x+4)$

How to solve a Rational Equation

1. Factor each denominator if possible
2. Find the LCD for all the denominators.
3. Multiply every term in the equation by the LCD. Cancel and reduce. This will leave you with an equation without fractions.
4. Solve the equation.
5. Check your solution. If the solution makes any of the denominators of the original rational equation zero then they cannot be used and there is no solution or ∅

Example 1

Solve for x

$$\frac{2}{3x} + \frac{1}{6} = \frac{3}{2x}$$

the LCD is $6x$ so
mutiply each term of
the equation by 6x

$$6x\left[\frac{2}{3x} + \frac{1}{6} = \frac{3}{2x}\right]$$

$$6x \bullet \frac{2}{3x} + 6x \bullet \frac{1}{6} = 6x \bullet \frac{3}{2x}$$

$$\cancel{6}^{2}\cancel{x} \bullet \frac{2}{\cancel{3}\cancel{x}} + \cancel{6}x \bullet \frac{1}{\cancel{6}} = \cancel{6}^{2}\cancel{x} \bullet \frac{3}{\cancel{2}\cancel{x}}$$

$4 + x = 6$

$x = 2$
(2 does not cause 0 in denominator so)

x = 2

Example 2

Solve for x

$$\frac{1}{6x} - \frac{1}{4} = \frac{-1}{3x}$$

the LCD is $12x$ so
mutiply each term of
the equation by 12x

$$12x\left[\frac{1}{6x} - \frac{1}{4} = \frac{-1}{3x}\right]$$

$$12x \bullet \frac{1}{6x} - 12x \bullet \frac{1}{4} = 12x \bullet \frac{-1}{3x}$$

$$\cancel{12}^{2}\cancel{x} \bullet \frac{1}{\cancel{6}\cancel{x}} + \cancel{12}^{3}x \bullet \frac{1}{\cancel{4}} = \cancel{12}^{4}\cancel{x} \bullet \frac{-1}{\cancel{3}\cancel{x}}$$

$2 + 3x = -4$
$3x = -6$
$x = -2$
(–2 does not cause 0 in denominator so)

$x = -2$

Example 3

Solve for x

$$\frac{x-4}{3x} + \frac{1}{2} = \frac{3}{4x}$$

the LCD is $12x$ so
mutiply each term of
the equation by 6x

$$12x\left[\frac{x-4}{3x} + \frac{1}{2} = \frac{3}{4x}\right]$$

$$12x \bullet \frac{x-4}{3x} + 12x \bullet \frac{1}{2} = 12x \bullet \frac{3}{4x}$$

$$\cancel{12}^{4}\cancel{x} \bullet \frac{x-4}{\cancel{3}\cancel{x}} + \cancel{12}^{6}x \bullet \frac{1}{\cancel{2}} = \cancel{12}^{3}\cancel{x} \bullet \frac{3}{\cancel{4}\cancel{x}}$$

$$4(x-4) + 6x = 9$$
$$4x - 16 + 6x = 9$$
$$10x = 25$$
$$x = \frac{25}{10} = \frac{5}{2}$$

$\frac{5}{2}$ does not cause 0 in denominator so

$$x = \frac{5}{2}$$

Example 4

Solve for x

$$\frac{1}{6} - \frac{x-5}{3x} = \frac{x+5}{4x}$$

the LCD is $12x$ so
mutiply each term of
the equation by 12x

$$12x\left[\frac{1}{6} - \frac{x-5}{3x} = \frac{x+5}{4x}\right]$$

$$12x \bullet \frac{1}{6} - 12x \bullet \frac{x-5}{3x} = 12x \bullet \frac{x+5}{4x}$$

$$\cancel{12}^{2}x \bullet \frac{1}{\cancel{6}} - \cancel{12}^{4}\cancel{x} \bullet \frac{x-5}{\cancel{3}\cancel{x}} = \cancel{12}^{3}\cancel{x} \bullet \frac{x+5}{\cancel{4}\cancel{x}}$$

$$2x - 4(x-5) = 3(x+5)$$
$$2x - 4x + 20 = 3x + 15$$
$$-2x + 20 = 3x + 15$$
$$5 = 5x$$
$$1 = x$$

1 does not cause 0 in denominator so

$$1 = x$$

Example 5

Solve for x

$$\frac{x}{x-2} - 4 = \frac{2}{x-2}$$

the LCD is $(x-2)$ so
mutiply each term of
the equation by $(x-2)$

$$(x-2)\left[\frac{x}{x-2} - 4 = \frac{2}{x-2}\right]$$

$$(x-2)\bullet\frac{x}{x-2} - (x-2)\bullet 4 = (x-2)\bullet\frac{2}{x-2}$$

$x-4(x-2)=2$
$x-4x+8=2$
$-3x=-6$
$x=2$
2 does cause 0 in denominator so

$\varnothing$ or No Solution

Example 6

Solve for x

$$\frac{x-6}{x(x-3)} = \frac{-2}{2(x-3)}$$

the LCD is $2x(x-3)$ so
mutiply each term of
the equation by $2x(x-3)$

$$2x(x-3)\left[\frac{x-6}{x(x-3)} = \frac{-2}{2(x-3)}\right]$$

$$2x(x-3)\bullet\frac{x-6}{x(x-3)} = 2x(x-3)\bullet\frac{-2}{2(x-3)}$$

$2(x-6)=-2x$
$2x-12=-2x$
$4x=12$
$x=3$
3 does cause 0 in denominator so

$\varnothing$ or No Solution

Example 7

Solve for x

$$\frac{3}{x+1} = \frac{1}{x-1} - \frac{2}{x^2-1}$$

factor each denominator

$$\frac{3}{x+1} = \frac{1}{x-1} - \frac{2}{(x+1)(x-1)}$$

the LCD is $(x+1)(x-1)$ so
mutiply each term of
the equation by $(x+1)(x-1)$

$$(x+1)(x-1)\left[\frac{3}{x+1} = \frac{1}{x-1} - \frac{2}{(x+1)(x-1)}\right]$$

$$(x+1)(x-1) \bullet \frac{3}{x+1} = (x+1)(x-1) \bullet \frac{1}{x-1} - (x+1)(x-1) \bullet \frac{2}{(x+1)(x-1)}$$

$3(x+1) = 1(x+1) - 2$

$3x + 3 = x + 1 - 2$

$3x + 3 = x - 1$

$2x = -4$

$x = 2$

2 does not cause 0 in denominator so

$x = 2$

Example 8

Solve for x

$$\frac{2x+3}{3x-9} - \frac{x}{2x-6} = \frac{4}{3}$$

factor each denominator

$$\frac{2x+3}{3(x-3)} - \frac{x}{2(x-3)} = \frac{4}{3}$$

the LCD is $6(x-3)$ so
mutiply each term of
the equation by $6(x-3)$

$$6(x-3)\left[\frac{2x+3}{3(x-3)} - \frac{x}{2(x-3)} = \frac{4}{3}\right]$$

$$\not{6}^{2}(x-3) \bullet \frac{2x+3}{\not{3}(x-3)} - 6(x-3) \bullet \frac{x}{(x-3)} = \not{6}^{2}(x-3) \bullet \frac{4}{\not{3}}$$

$2(2x+3) - 6x = 8(x-3)$
$4x + 6 - 6x = 8x - 24$
$-2x + 6 = 8x - 24$
$30 = 10x$
$x = 3$
3 does cause 0 in denominator so

$\varnothing$ or No Solution