Section 5 – 3: Simplifying Complex Rational Expressions

A complex rational expression is a fraction that has fractions in the numerator and/or denominator.

Examples of Complex Rational Expressions

$$\frac{\frac{2}{x}+\frac{3}{x+1}}{\frac{2}{x}-\frac{3}{4}} \qquad \frac{5-\frac{2}{x-7}}{\frac{5}{x-8}-3x} \qquad \frac{7}{\frac{2}{x}+\frac{4}{x-5}}$$

n previous sections we solved equations that had fractions with whole numbers for the denominators. In that section we solved the equations by first using the multiplication property of equality to multiply **every term in an equation** by the Least Common Denominator. When we multiply each term by the LCD the resulting equation will not have any fractions in it. We can use a similar technique to simplify the complex fractions

How to Simplify A Complex Rational Expressions

1. Factor each denominator if possible.
2. Find the LCD for all the denominators.
3. Multiply every term in the numerator and denominator by the LCD. This will leave you with a polynomial without fractions in the numerator or the denominator.
4. Simplify the numerator and the denominators separately by combining like terms.
5. Factor and reduce if possible. This step is not common but could happen.

Example 1

$$\frac{\frac{2}{x}+\frac{5}{6}}{\frac{2}{x}-\frac{3}{4}}$$ multiply all 4 terms by 12x

$$\frac{\left(\frac{12x}{1}\right)\frac{2}{x}+\frac{5}{6}\left(\frac{12x}{1}\right)}{\left(\frac{12x}{1}\right)\frac{2}{x}-\frac{3}{4}\left(\frac{12x}{1}\right)}$$

$$\frac{24+10x}{24-9x}$$ Try to reduce by factoring

$$\frac{2(5x+12)}{-3(3x-8)}$$

Example 2

$$\frac{\frac{4}{3}-\frac{2}{x+1}}{\frac{3}{x+1}+\frac{3}{2}}$$ multiply all 4 terms by $6(x+1)$

$$\frac{\left(\frac{6(x+1)}{1}\right)\frac{4}{3}-\frac{2}{x+1}\left(\frac{6(x+1)}{1}\right)}{\left(\frac{6(x+1)}{1}\right)\frac{3}{x+1}+\frac{3}{2}\left(\frac{6(x+1)}{1}\right)}$$

$$\frac{8(x+1)-12}{18+9(x+1)}$$ simplify numerator and denominator

$$\frac{8x-4}{9x+27}$$ Try to reduce by factoring

$$\frac{4(x-1)}{9(x+3)}$$

Example 3

$$\frac{\frac{3}{4x^2}-\frac{2}{x}}{\frac{1}{3x}+\frac{5}{6}}$$ multiply all 4 terms by $12x^2$

$$\frac{\left(\frac{12x^2}{1}\right)\frac{3}{4x^2}-\frac{2}{x}\left(\frac{12x^2}{1}\right)}{\left(\frac{12x^2}{1}\right)\frac{1}{3x}+\frac{5}{6}\left(\frac{12x^2}{1}\right)}$$

$$\frac{9-24x}{4x+10x^2}$$ simplify numerator and denominator

$$\frac{-3(8x+3)}{2x(5x+2)}$$ Try to reduce by factoring

$$\frac{-3(8x+3)}{2x(5x+2)}$$

Example 3

$$\frac{\frac{2}{x+1}\left(\frac{(x+1)(x-1)}{1}\right)}{\frac{3}{x-1}\left(\frac{(x+1)(x-1)}{1}\right)+\frac{4}{x+1}\left(\frac{(x+1)(x-1)}{1}\right)}$$ multiply all 3 terms by $(x+1)(x-1)$

$$\frac{2(x-1)}{3(x+1)+4(x-1)}$$

$$\frac{2x-2}{3x+3+4x-4}$$ simplify numerator and denominator

$\frac{2x-2}{7x-1}$ Try to reduce by factoring

$$\frac{2(x-1)}{(7x-1)}$$

Example 4

$$\frac{\dfrac{1}{2x+6}}{\dfrac{2}{x^2-9}-\dfrac{5}{x-3}}$$ factor the numerator and denominator

$$\frac{\dfrac{1}{2(x+3)}}{\dfrac{2}{(x+3)(x-3)}-\dfrac{5}{x-3}}$$

$$\frac{\dfrac{1}{2(x+3)}}{\dfrac{2}{(x+3)(x-3)}-\dfrac{5}{x-3}}$$ multiply all 4 terms by $2(x+3)(x-3)$

$$\frac{\left(\dfrac{2(x+3)(x-3)}{1}\right)\dfrac{1}{2(x+3)}}{\left(\dfrac{(2)(x+3)(x-4)}{1}\right)\dfrac{2}{(x+3)(x-3)}-\dfrac{5}{x-3}\left(\dfrac{(2)(x+3)(x-3)}{1}\right)}$$

$$\frac{x-3}{4-10(x+3)}$$ simplify numerator and denominator

$$\frac{x-3}{-10x-27}$$ Cannot reduce by factoring

