

Section 5 – 2B: Adding and Subtracting Rational Expressions

Adding and Subtracting Fractions without a Common Denominator

To add or subtract fractions, each fraction **MUST** have the same denominator. If the fractions do not have a common denominator you must make new fractions that both have a common denominator. To make each fraction's denominator into the LCD, multiply the top and bottom of each fraction by the fraction that will cause each denominator to become the LCD.

Example 1

Add

$$\frac{7}{4} + \frac{5}{6}$$

The LCD is 12

multiply $\frac{7}{4}$ by $\frac{3}{3}$ to get an LCD of $\frac{\quad}{12}$

multiply $\frac{5}{6}$ by $\frac{2}{2}$ to get an LCD of $\frac{\quad}{12}$

$$\frac{3 \cdot 7}{4 \cdot 3} + \frac{2 \cdot 5}{2 \cdot 6}$$

$$\frac{21}{12} + \frac{10}{12}$$

$$= \frac{21+10}{12} \quad \begin{array}{l} \text{add the numerators} \\ \text{over the common denominator} \end{array}$$

combine like terms in the numerator

$$= \frac{31}{12}$$

Example 2

Add

$$\frac{x-1}{4x} + \frac{x+2}{10x}$$

The LCD is $20x$

multiply $\frac{x-1}{4x}$ by $\frac{5}{5}$ to get an LCD of $\frac{\quad}{20x}$

multiply $\frac{x+2}{9x}$ by $\frac{2}{2}$ to get an LCD of $\frac{\quad}{20x}$

$$\frac{5 \cdot x-1}{5 \cdot 4x} + \frac{2 \cdot x+2}{2 \cdot 10x}$$

$$\frac{5x-5}{20x} + \frac{2x+4}{20x}$$

$$= \frac{5x-5+2x+4}{20x} \quad \begin{array}{l} \text{add the numerators} \\ \text{over the common denominator} \end{array}$$

combine like terms in the numerator

$$= \frac{7x-1}{20x}$$

Example 3

Add

$$\frac{2x-1}{6(x+4)} + \frac{x-3}{15(x+4)}$$

The LCD is $30(x+4)$

multiply $\frac{2x-1}{6(x+4)}$ by $\frac{5}{5}$ to get an LCD of $\frac{\quad}{30(x+4)}$

multiply $\frac{x-3}{15(x+4)}$ by $\frac{2}{2}$ to get an LCD of $\frac{\quad}{30(x+4)}$

$$\frac{5}{5} \cdot \frac{2x-1}{6(x+4)} + \frac{2}{2} \cdot \frac{x-3}{15(x+4)}$$

$$\frac{10x-5}{30(x+4)} + \frac{2x-6}{30(x+4)}$$

$$= \frac{10x-5+2x-6}{12(x+4)} \quad \begin{array}{l} \text{add the numerators} \\ \text{over the common denominator} \end{array}$$

combine like terms in the numerator

$$= 30 \frac{12x-11}{12(x+4)}$$

Example 4

Add

$$\frac{6}{x-2} + \frac{3x+5}{x^2-4} \text{ factor each denominator}$$

$$\frac{6}{(x-2)} + \frac{3x+5}{(x-2)(x-2)}$$

The LCD is $(x-2)(x-2)$

$$\text{multiply } \frac{6}{(x-2)} \text{ by } \frac{(x-2)}{(x-2)} \text{ to get an LCD of } \frac{\quad}{(x-2)(x-2)}$$

$$\text{multiply } \frac{3x+5}{(x-2)(x-2)} \text{ by } \frac{1}{1} \text{ to get an LCD of } \frac{\quad}{(x-2)(x-2)}$$

$$\frac{(x-2)}{(x-2)} \cdot \frac{6}{(x-2)} + \frac{1}{1} \cdot \frac{3x+5}{(x-2)(x-2)}$$

$$\frac{6x-12}{(x-2)(x-2)} + \frac{3x+5}{(x-2)(x-2)}$$

$$= \frac{6x-12+3x+5}{12(x+4)} \quad \begin{array}{l} \text{add the numerators} \\ \text{over the common denominator} \end{array}$$

combine like terms in the numerator

$$= \frac{9x-7}{12(x+4)}$$

Example 5

Add

$$\frac{x+3}{4x-20} + \frac{5}{12} \text{ factor each denominator}$$

$$\frac{x+3}{4(x-5)} + \frac{5}{12}$$

The LCD is $12(x-5)$

multiply $\frac{x+3}{4(x-5)}$ by $\frac{3}{3}$ to get an LCD of $\frac{\quad}{12(x-5)}$

multiply $\frac{5}{12}$ by $\frac{(x-5)}{(x-5)}$ to get an LCD of $\frac{\quad}{12(x-5)}$

$$\frac{3}{3} \cdot \frac{x+3}{4(x-5)} + \frac{(x-5)}{(x-5)} \cdot \frac{5}{12}$$

$$\frac{3x+9}{12(x-5)} + \frac{5x-25}{12(x-5)}$$

$$= \frac{3x+9+5x-25}{12(x-5)} \quad \begin{array}{l} \text{add the numerators} \\ \text{over the common denominator} \end{array}$$

combine like terms in the numerator

$$= \frac{8x-15}{12(x-5)}$$

Example 6

Subtract

$$\frac{2x-1}{9(x+4)} - \frac{3x-2}{6(x+4)}$$

The LCD is $18(x+4)$

multiply $\frac{2x-1}{9(x+4)}$ by $\frac{2}{2}$ to get an LCD of $\frac{\quad}{18(x+4)}$

multiply $\frac{3x-2}{6(x+4)}$ by $\frac{3}{3}$ to get an LCD of $\frac{\quad}{18(x+4)}$

$$\frac{2}{2} \cdot \frac{2x-1}{9(x+4)} - \frac{3}{3} \cdot \frac{3x-2}{6(x+4)}$$

$$\frac{4x+2}{12(x-5)} - \frac{9x-6}{12(x-5)}$$

$$= \frac{4x+2-(9x-6)}{12(x-5)} \quad \begin{array}{l} \text{subtract the numerators} \\ \text{over the LCD} \end{array}$$

$$\frac{4x+2-(9x-6)}{12(x-5)} \quad \text{distribute the } -$$

$$\frac{4x+2-9x+6}{12(x-5)}$$

combine like terms in the numerator

$$= \frac{-5x+7}{12(x-5)}$$

Example 7

Subtract

$$\frac{4}{3x-6} - \frac{2}{3x} \text{ factor each denominator}$$

$$\frac{4}{3(x-2)} - \frac{2}{3x}$$

The LCD is $3x(x-2)$

multiply $\frac{4}{3(x-2)}$ by $\frac{x}{x}$ to get an LCD of $\frac{\quad}{3x(x-2)}$

multiply $\frac{2}{3x}$ by $\frac{(x-2)}{(x-2)}$ to get an LCD of $\frac{\quad}{3x(x-2)}$

$$\frac{x}{x} \cdot \frac{4}{3(x-2)} - \frac{(x-2)}{(x-2)} \cdot \frac{2}{3x}$$

$$\frac{4x}{3x(x-2)} - \frac{2x-4}{3x(x-2)}$$

$$= \frac{4x - (2x - 4)}{3x(x-2)} \text{ subtract the numerators}$$

over the LCD

$$\frac{4x - (2x - 4)}{3x(x-2)} \text{ distribute the -}$$

$$\frac{4x - 2x + 8}{3x(x-2)}$$

combine like terms in the numerator

$$= \frac{2x + 8}{3x(x-2)}$$

Example 8

Subtract

$$\frac{4}{x+1} - \frac{2}{x-2}$$

The LCD is $(x-1)(x-2)$

multiply $\frac{4}{x+1}$ by $\frac{(x-2)}{(x-2)}$ to get an LCD of $\frac{\quad}{(x-1)(x-2)}$

multiply $\frac{2}{x-2}$ by $\frac{(x+1)}{(x+1)}$ to get an LCD of $\frac{\quad}{(x-1)(x-2)}$

$$\frac{(x-2) \cdot 4}{(x-2)(x+1)} - \frac{(x+1) \cdot 2}{(x+1)(x-2)}$$

$$\frac{4-8x}{(x-1)(x-2)} - \frac{2x+2}{(x-1)(x-2)}$$

$$= \frac{4x-8-(2x+2)}{(x-1)(x-2)} \quad \begin{array}{l} \text{subtract the numerators} \\ \text{over the LCD} \end{array}$$

$$\frac{4x-8-(2x+2)}{(x-1)(x-2)} \quad \text{distribute the } -$$

$$\frac{4x-8-2x-2}{(x-1)(x-2)}$$

combine like terms in the numerator

$$= \frac{2x-10}{(x-1)(x-2)} \quad \text{Factor the numerator}$$

$$= \frac{2(x-5)}{(x-1)(x-2)} \quad \text{Does not reduce}$$

Example 9

Subtract

$$\frac{4}{3x+3} - \frac{x-4}{2x^2-2} \text{ factor each denominator}$$

$$\frac{4}{3(x+1)} - \frac{x-4}{2(x-1)(x+1)}$$

The LCD is $6(x+1)(x-1)$

multiply $\frac{4}{3(x+1)}$ by $\frac{2(x-1)}{2(x-1)}$ to get an LCD of $\frac{\quad}{6(x+1)(x-1)}$

multiply $\frac{x-4}{2(x-1)(x+1)}$ by $\frac{3}{3}$ to get an LCD of $\frac{\quad}{6(x+1)(x-1)}$

$$\frac{2(x-1)}{2(x-1)} \cdot \frac{4}{3(x+1)} - \frac{3}{3} \cdot \frac{x-4}{2(x-1)(x+1)}$$

$$\frac{8x-8}{6(x+1)(x-1)} - \frac{3x-12}{6(x+1)(x-1)}$$

$$= \frac{8x-8-(3x-12)}{6(x+1)(x-1)} \text{ subtract the numerators}$$

over the LCD

$$= \frac{8x-8-(3x-12)}{6(x+1)(x-1)} \text{ distribute the -}$$

$$= \frac{8x-8-3x+12}{6(x+1)(x-1)}$$

combine like terms in the numerator

$$= \frac{5x+4}{6(x+1)(x-1)}$$