

Section 5–1B: Multiplying or Dividing Rational Expressions

Multiplying Fractions

The basic rule for multiplying fractions is to multiply the numerators together and then multiply the denominators together

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

and then reduce the answer.

It is often much faster if we first reduce the fractions first. This is done by factoring each part and then canceling common factors.

Example 1

$$\begin{aligned} & \frac{6}{5} \cdot \frac{15}{4} \\ &= \frac{2 \cdot 3}{5} \cdot \frac{3 \cdot 5}{2 \cdot 2} \quad \begin{array}{l} \text{factor each numerator} \\ \text{factor each denominator} \end{array} \\ &= \frac{\cancel{2} \cdot 3}{\cancel{5}} \cdot \frac{3 \cdot \cancel{5}}{\cancel{2} \cdot 2} \quad \text{cancel all common factors} \\ &= \frac{9}{2} \end{aligned}$$

Example 2

$$\begin{aligned} & \frac{21}{14} \cdot \frac{15}{33} \\ &= \frac{7 \cdot 3}{7 \cdot 2} \cdot \frac{3 \cdot 5}{3 \cdot 11} \quad \begin{array}{l} \text{factor each numerator} \\ \text{factor each denominator} \end{array} \\ &= \frac{\cancel{7} \cdot 3}{\cancel{7} \cdot 2} \cdot \frac{3 \cdot 5}{\cancel{3} \cdot 11} \quad \text{cancel all common factors} \\ &= \frac{15}{22} \end{aligned}$$

Multiplying Rational Expressions

We multiply Rational Expressions the same way we multiply any fraction. To do this we first **completely factor** all the numerators and denominators and then cancel out all the common factors.

- 1, **Completely factor all the** the numerators and denominators.
2. Cancel out the common factors.
3. Write the remaining factors in fraction form

Example 3

$$\begin{aligned} & \frac{x^2 - 4}{20x - 8} \cdot \frac{4x - 12}{15x + 6} \\ &= \frac{(x-2)(x+2)}{x-3} \cdot \frac{4(x-3)}{5(x+2)} \quad \begin{array}{l} \text{factor each numerator} \\ \text{factor each denominator} \end{array} \\ & \quad \text{cancel all common factors} \\ &= \frac{4(x-2)}{5} \end{aligned}$$

Example 4

$$\begin{aligned} & \frac{3x-3}{x^2} \cdot \frac{5x}{6x-6} \\ &= \frac{3(x-1)}{x^2} \cdot \frac{5x}{6(x-1)} \quad \begin{array}{l} \text{factor each numerator} \\ \text{factor each denominator} \end{array} \end{aligned}$$

cancel all common factors

$$= \frac{5}{2x}$$

Example 5

$$\begin{aligned} & \frac{4x-4}{x^2-2x-8} \cdot \frac{x-4}{x^2-1} \\ &= \frac{4(x-1)}{(x+2)(x-4)} \cdot \frac{x-4}{(x+1)(x-1)} \quad \begin{array}{l} \text{factor each numerator} \\ \text{factor each denominator} \end{array} \end{aligned}$$

cancel all common factors

$$= \frac{4}{(x+2)(x+1)}$$

Example 6

$$\begin{aligned} &= \frac{4x^2y^3}{5x-15} \cdot \frac{x^2-9}{12x^4y} \\ &= \frac{4x^2y^3}{5(x-3)} \cdot \frac{(x+3)(x-3)}{12x^4y} \quad \begin{array}{l} \text{factor each numerator} \\ \text{factor each denominator} \end{array} \end{aligned}$$

cancel all common factors

$$= \frac{y^2(x+3)}{15x^2}$$

Dividing Rational Expressions

We divide Rational Expressions like

$$\frac{a}{b} \div \frac{c}{d}$$

by changing the division problem into a multiplication problem. We do this by inverting (flipping over) the second fraction and change the problem to multiplication.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

1. Invert the second fraction and change the problem to multiplication.
2. **Completely factor all the** the numerators and denominators.
3. Cancel out the common factors.
4. Write the remaining factors in fraction form

Example 7

$$\frac{3x-15}{21x^5} \div \frac{x^2-25}{14x^2} \quad \text{Invert the second fraction and multiply}$$

$$= \frac{3x-15}{21x^5} \cdot \frac{14x^2}{x^2-25}$$

$$= \frac{3(x-5)}{21x^5} \cdot \frac{14x^2}{(x+5)(x-5)} \quad \begin{array}{l} \text{factor each numerator} \\ \text{factor each denominator} \end{array}$$

cancel all common factors

$$= \frac{2}{x^3(x+5)}$$

Example 8

$$\frac{3x+6}{x^2-9} \div \frac{5x+10}{x-3} \quad \text{Invert the second fraction and multiply}$$

$$= \frac{3x-6}{2x-6} \cdot \frac{x-3}{5x+10}$$

$$= \frac{3(x+2)}{(x+3)(x-3)} \cdot \frac{(x-3)}{5(x+2)} \quad \begin{array}{l} \text{factor each numerator} \\ \text{factor each denominator} \end{array}$$

cancel all common factors

$$= \frac{3}{(x+3)}$$

Example 9

$$\frac{x^2-x-6}{x^2+3x+2} \div \frac{5x-15}{x^2+2x} \quad \text{Invert the second fraction and multiply}$$

$$= \frac{x^2-x-6}{x^2+3x+2} \cdot \frac{x^2+2x}{5x-15}$$

$$= \frac{(x-3)(x+2)}{(x+2)(x+1)} \cdot \frac{x(x+2)}{5(x-3)} \quad \begin{array}{l} \text{factor each numerator} \\ \text{factor each denominator} \end{array}$$

cancel all common factors

$$= \frac{x(x+2)}{(x+1)}$$