Section 5 – 1A: Reducing Rational Expressions

Rational Expressions

A rational number is a number that can be written as the ratio of two rational numbers. We often think of a rational numbers as a fraction with integers for the numerator and denominator. We can expend this concept to include algebraic expressions by allowing the numerator and denominator of the fraction to be any polynomial expression. This expended form is called a Rational Expression.

Rational Expression: A fraction with a polynomial expression in the numerator and denominator.

Example 1

$$\frac{2x^2 - 10x}{4x}$$

Example 2

$$\frac{8x^2 - 6x}{x - 8}$$

Example 3

$$\frac{5x^2 - 25x}{x^2 - 3x - 10}$$

Note: The denominator of a fraction cannot have a value of zero. When we have a polynomial in the denominator any value of the variable that would make the denominator zero is not allowed. This restriction will not be explored in the first part of the chapter. We will revisit it in the last section.

Example 4

$$\frac{2x^2 - 10x}{4x} \;;\; x \neq 0$$

Example 5

$$\frac{8x^2 - 6x}{x + 8} \;;\; x \neq -8$$

Example 6

$$\frac{5x}{2x - 3} \;;\; x \neq \frac{3}{2}$$

Reducing a Fraction to Lowest Terms

We reduce or simplify a fraction by canceling out all the common factors. To do this we fist **completely factor** the numerator and denominator and then cancel out all the common factors.

1, **Completely factor** the numerator and denominator.

2. Cancel out the common factors.

Example 7

$$\frac{15}{20}$$
$$= \frac{3 \bullet 5}{5 \bullet 2 \bullet 2}$$
$$= \frac{3 \bullet \cancel{5}}{\cancel{5} \bullet 2 \bullet 2}$$
$$= \frac{3}{4}$$

Example 8

$$\frac{21}{6}$$
$$= \frac{3 \bullet 7}{2 \bullet 3}$$
$$= \frac{\cancel{3} \bullet 7}{2 \bullet \cancel{3}}$$
$$= \frac{7}{2}$$

Example 9

$$\frac{20}{36}$$
$$= \frac{5 \bullet 2 \bullet 2}{3 \bullet 3 \bullet 2 \bullet 2}$$
$$= \frac{5 \bullet \cancel{2} \bullet \cancel{2}}{3 \bullet 3 \bullet \cancel{2} \bullet \cancel{2}}$$
$$= \frac{5}{9}$$

A **Rational Expression** is a fraction with a polynomial expression in the numerator and denominator. A **polynomial** is an expression that has two or more terms each separated by a + or − sign. This means that both the numerator and denominator will consist of the addition or subtraction of several separate terms.

You cannot cancel any part of a polynomial expression in the numerator with part of a polynomial expression in the denominator.

We know from PEMDAS that the following is true.

$$\frac{6+2}{4}=\frac{8}{4}=2$$

you cannot cancel the 6 or the 2 in the numerator with the 4 in the denominator. It will not result in a true statement.

$$\frac{6+\not{2}^{1}}{\not{4}^{2}}=\frac{6+1}{2}=\frac{7}{2} \quad \text{which is not 2}$$

$$\frac{\not{6}^{3}+2}{\not{4}^{2}}=\frac{3+2}{2}=\frac{5}{2} \quad \text{which is not 2}$$

You CANNOT cancel any part of a polynomial expression. You cannot cancel part of an addition or subtraction expression.

Example 10

$$\frac{5x+6}{3}$$

You **cannot** cancel the 6 and the 3

Example 11

$$\frac{4x^2+2x-7}{6x}$$

You **cannot** cancel the $4x^2$ and the $6x$ or the $2x$ and the $6x$

Example 12

$$\frac{4x^2+5x-3}{4x^2-9}$$

You **cannot** cancel the $4x^2$ and the $4x^2$ or the –3 and the –9

When you factor the polynomial expression into separate factors you CAN then cancel common factors.

Example 13

$$\frac{6x(x-3)}{12(x-9)}$$

the $6x$, $(x-3)$, 12 and $(x-9)$ are each factors or products so **you can cancel** the 6 in the numerator with the 12 in the denominator

Example 14

$$\frac{(x-2)(x+3)}{3(x-2)}$$

the $(x-2)$,$(x+3)$, 3 and t $(x-2)$ are each factors or products so **you can cancel** the $(x-2)$ in the numerator with the $(x-2)$ in the denominator

Reducing Rational Expressions

We reduce or simplify a **Rational Expression** by canceling out all the common factors. To do this we fist **completely factor** the numerator and denominator and then cancel out all the common factors.

1, **Completely factor** the numerator and denominator.

2. Cancel out the common factors.

Simplify each expression.

Example 15

$$\frac{x^2-16}{2x^2-8x}$$ factor the numerator / factor the denominator

$$=\frac{(x-4)(x+4)}{2x(x-4)}$$ cancel all common factors

$$=\frac{(x+4)}{2x}$$

Example 16

$$\frac{x^2-6x+5}{x^2-25}$$ factor the numerator / factor the denominator

$$=\frac{(x-1)(x-5)}{(x-5)(x+5)}$$ cancel all common factors

$$=\frac{(x-1)}{(x+5)}$$

Example 17

$$\frac{10x^2-15x}{4x^2-9}$$ factor the numerator / factor the denominator

$$=\frac{5x(2x-3)}{(2x-3)(2x+3)}$$ cancel all common factors

$$=\frac{5x}{(2x+3)}$$

Example 18

$$\frac{x^2-x-6}{x^2-6x+9}$$ factor the numerator / factor the denominator

$$=\frac{(x-3)(x+2)}{(x-3)(x-3)}$$ cancel all common factors

$$=\frac{(x+2)}{(x-3)}$$

Example 19

$$\frac{3x^2-6x}{4x^3-8x^2}$$ factor the numerator / factor the denominator

$$=\frac{3x(x-2)}{4x^2(x-2)}$$ cancel all common factors

$$=\frac{3}{4x}$$

Example 20

$$\frac{6xy^2-3x^2y}{12xy-6x^2}$$ factor the numerator / factor the denominator

$$=\frac{3xy(2y-x)}{6x(2y-x)}$$ cancel all common factors

$$=\frac{y}{2}$$

Example 21

$$\frac{5x-10}{4-x^2}$$ factor out a -1

$$\frac{5x-10}{-(x^2-4)}$$ factor the numerator / factor the denominator

$$=\frac{5(x-2)}{-(x+2)(x-2)}$$ cancel all common factors

$$=\frac{5}{-(x+2)}$$

Example 22

$$\frac{16-x^2}{3x^2-12x}$$ factor out a -1

$$\frac{-(x^2-16)}{3x^2-12x}$$ factor the numerator / factor the denominator

$$=\frac{-(x-4)(x+4)}{3x(x-4)}$$ cancel all common factors

$$=\frac{-(x+4)}{3x}$$

Note: In past chapters, if we had a fraction as an answer with a negative sign in the denominator such as $\frac{5}{-2}$ we would rewrite it as $\frac{-5}{2}$. We will chose to not follow that practice in the chapter but you could if you desired. That would change the answer to Example 16 from $\frac{5}{-(x+2)}$ to $\frac{-5}{(x+2)}$

