Tool 1

Greatest Common Factor (GCF)

This is a very important tool. **You must try to factor out the GCF first** in **every problem.** Some problems do not have a GCF but many do. When you factor out the GCF you may recognize the **expression inside the parentheses** as a term that can be factored further by one of the other types of factoring tools.

Example 1

Factor:

$5x^2y + 20xy - 5y$

(the GCF is 5y)

$$\frac{5x^2y}{5y} + \frac{20xy}{5y} - \frac{5y}{5y}$$

$$= 5y(x^2 + 4x - 1)$$

Example 2

Factor:

$10x^3 - 20x^2 + 50x$

(the GCF is $10x$)

$$\frac{10x^3}{10x} - \frac{20x^2}{10x} + \frac{50x}{10x}$$

$$= 10x(x^2 - 2x + 5)$$

Tool 2

Factor By Grouping Method

Factoring by Grouping requires **four terms**. You **group the first two terms together** and **the last two terms together** and then **factor out the GCF in the first group** and **factor out the GCF in the second group.** The final step requires that you take out the **common binomial** in each term. You will then have the final factored form which is the product of two binomials.

Example 3

Factor:

$6x^2 - 4x - 15x + 10$

$2x(3x - 2) - 5(3x - 2)$

$(3x - 2)(2x - 5)$

Example 4

Factor:

$20x^2 + 12x - 5x - 3$

$4x(5x + 3) - 1(5x + 3)$

$(5x + 3)(4x - 1)$

Tool 3A

Factoring The Difference of Two Perfect Squares

This tool requires **Two Terms that are Perfect Squares separated by a – sign.**

The factors of The Difference of Two Perfect Squares are
the product of the sum and difference of the factors
that make up each of the perfect squares.

Example 5

Factor:

$$4x^2 - 25$$

$$4x^2 = 2x \bullet 2x \text{ and } 25 = 5 \bullet 5$$

$$= (2x+5)(2x-5)$$

Example 6

Factor:

$$9x^2 - 25y^2$$

$$9x^2 = 3x \bullet 3x \text{ and } 5y^2 = 5y \bullet 5y$$

$$= (3x+5y)(3x-5y)$$

Tool 3B

The SUM of Two Perfect Squares

This tool requires **Two Terms that are Perfect Squares separated by a + sign.**

The **Sum of Two Perfect Squares Does NOt Factor (DNF)**

Example 7

$$9x^2 + 16$$

Does not factor

DNF

Example 8

$$4x^2 + 25$$

Does not factor

DNF

Tool 4A

Factoring The Difference of Two Perfect Cubes

This tool requires **Two Terms that are Perfect Cubes separated by a – sign.**

Factor: $a^3 - b^3$

$= (a \quad - \quad b) \quad (a^2 \ + \ ab \ + \ b^2)$

$(a-b)$ is found by $\begin{pmatrix} \text{perfect cube root} & \text{keep the} & \text{perfect cube root} \\ \text{of the first term} & \text{same sign} & \text{of the last term} \end{pmatrix}$

$(a \quad - \quad b) \quad (a^2 \ + \ ab \ + \ b^2)$ is found by

$(\text{First} - \text{ Last}) \ \left(\text{First}^2 \ \text{ change sign } \ \text{First} \bullet \text{Last} \ \ \text{ always} + \ \text{Last}^2\right)$

Example 9

Factor: $8x^3 - 125$

$= (2x)^3 - 5^3$

$= (2x-5) \ \left(4x^2 + \ 10x \ + \ 25\right)$

$\quad \text{F} \ \ \text{L} \quad \text{F}^2 \text{ cs } \text{F} \bullet \text{L} \ + \ \text{L}^2$

Example 10

Factor: $64x^3 - 27$

$= (4x)^3 - 3^3$

$(4x-3) \ \left(4x^2 + \ 12x \ + \ 9\right)$

$\quad \text{F} \quad \text{L} \quad \text{F}^2 \text{ cs } \ \text{F} \bullet \text{L} \ + \ \text{L}^2$

Tool 4B

The SUM of Two Perfect Cubes

This tool requires **Two Terms that are Perfect Cubes separated by a + sign.**

Factoring The Sum of Two Perfect Cubes

Factor: $a^3 + b^3$

$= (a \;+\; b) \quad \left(a^2 \;-\; ab \;+\; b^2\right)$

$(a+b)$ is found by $\begin{pmatrix} \text{perfect cube root} & \text{keep the} & \text{perfect cube root} \\ \text{of the first term} & \text{same sign} & \text{of the last term} \end{pmatrix}$

$(a \;+\; b) \quad \left(a^2 \;-\; ab \;+\; b^2\right)$ is found by

$(\text{First} - \text{Last}) \left(\text{First}^2 \quad \text{change sign} \quad \text{First} \bullet \text{Last} \quad \text{always} + \ \text{Last}^2\right)$

Example 11

Factor: $27x^3 - 8$

$= (3x)^3 - 2^3$

$(3x+2)\left(9x^2 \;-\; 6x \;+\; 4\right)$

$\text{F} \quad \text{L} \quad \text{F}^2 \text{ cs F} \bullet \text{L} + \text{L}^2$

Example 12

Factor: $125x^3 + 64$

$= (5x)^3 + 4^3$

$(5x+4)\left(25x^2 - 20x + 16\right)$

$\text{F} \quad \text{L} \quad \text{F}^2 \text{ cs F} \bullet \text{L} + \text{L}^2$

Tool 5A

Factoring Easy Trinomials

that have $1x^2$ as the **first term** and

ends with a positive number (C is positive).

Factor

$$1x^2 \underline{+} Bx + C$$

into (x $\underline{+}$ D) (x $\underline{+}$ E)

D and E must MULTIPLY to + C
and ADD to $\underline{+}$ B

Example 13

Factor $x^2 - 9x + 20$

into (x $\underline{+}$ D) (x $\underline{+}$ E)

$1 \cdot 20$

$2 \cdot 10$

$4 \cdot 5$

$x^2 - 9x + 20$

we need two numbers
D and E that
multiply to + 20
and **add to – 9**

– 4 and **– 5** work

Answer: $(x-4)\ (x-5)$

Example 14

Factor $x^2 + 9x + 18$

into (x $\underline{+}$ D) (x $\underline{+}$ E)

$1 \cdot 18$

$2 \cdot 9$

$3 \cdot 6$

$x^2 + 9x + 18$

we need two numbers
D and E that
multiply to + 18
and **add to + 9**

+ 3 and **+ 6** work

Answer: $(x+3)\ (x+6)$

Example 15

Factor $x^2 - 7x + 12$

into (x $\underline{+}$ D) (x $\underline{+}$ E)

$1 \cdot 12$

$2 \cdot 6$

$3 \cdot 4$

$x^2 - 7x + 12$

we need two numbers
D and E that
multiply to + 12
and **add to – 7**

– 3 and **– 4** work

Answer: $(x-3)\ (x-4)$

Tool 5B

Factoring Easy Trinomials
that have $1x^2$ as the **first term** and
ends with a negative number (C is negative).

Factor
$1x^2 \pm Bx - C$
into (**x ± D**) (**x ± E**)

D and E must Multiply to – C
and SUBTRACT to ± B

Example 16

Factor $x^2 - x - 20$
into (**x ± D**) (**x ± E**)

$1 \cdot 20$
$2 \cdot 10$
$4 \cdot 5$

$x^2 - x - 20$

we need two numbers
D and **E** that
multiply to – 20
and **subtract to – 1**

– 5 and **4** work

Answer: $(x-5)\ (x+4)$

Example 17

Factor $x^2 + 3x - 18$
into (**x ± D**) (**x ± E**)

$1 \cdot 18$
$2 \cdot 9$
$3 \cdot 6$

$x^2 + 3x - 18$

we need two numbers
D and **E** that
multiply to – 18
and **subtract to + 3**

+ 6 and **– 3** work

Answer: $(x+6)\ (x-3)$

Example 18

Factor $x^2 - 4x - 12$
into (**x ± D**) (**x ± E**

$1 \cdot 12$
$2 \cdot 6$
$3 \cdot 4$

$x^2 - 4x - 12$

we need two numbers
D and **E** that
multiply to – 12
and **subtract to – 4**

+ 6 and **– 2** work

Answer: $(x+6)\ (x-2)$

Tool 6A

Factoring Hard Trinomials like $Ax^2 \pm Bx \pm C$ where $A > 1$

that END with a POSITIVE number (C is POSITIVE)

by

the Creating an Easy Trinomial Method

Example 19

Factor: $2x^2 - 9x + 10$

Step 1: The **GCF must be taken out first (if there is one)** before factoring the hard trinomial.

Step 2: Create an Easy Trinomial by moving the coefficient of the **2**x^2 term to the end of the trinomial and multiplying the **2** and the 10

$$2x^2 - 9x + 10 \bullet 2$$

to get the easy trinomial

$$x^2 - 9x + 20$$

Step 3: Factor the easy trinomial by finding the 2 numbers that **multiply to + 20 and add to –9**

– 5 and – 4

(x – 5) (x – 4)

Step 4: In step 1 you multiplied the constant 10 by the **2** that was the coefficient of the **2**x^2 term. Now **divide BOTH of the constants** in (x – 5)(x – 4) by **2**

$$\left(x - \frac{5}{2}\right)\left(x - \frac{4}{2}\right)$$

reduce each fraction

$$\left(x - \frac{5}{2}\right)(x - 2)$$

Step 5: "glide" the denominator of each fraction (if there is one) to the front of the x term

$$\left(2x - \frac{5}{2}\right)(x - 2)$$

Answer: $(2x - 5)(x - 2)$

Tool 6B

Factoring Hard Trinomials like $Ax^2 \pm Bx \pm C$ where $A > 1$

that END with a NEGATIVE number (C is NEGATIVE)

by

the Creating an Easy Trinomial Method

Example 20

Factor: $6x^2 - x - 1$

Step 1: The **GCF must be taken out first (if there is one)** before factoring the hard trinomial.

Step 2: Create an Easy Trinomial by moving the coefficient of the **6** x^2 term to the end of the trinomial and multiplying the **6** and the –1

$$6x^2 - x - 1 \bullet 6$$

to get the easy trinomial

$$x^2 - x - 6$$

Step 3: Factor the easy trinomial by finding the 2 numbers that **multiply to – 6 and subtract to – 1**

– 3 and + 2

(x – 3) (x + 2)

Step 4: In step 1 you multiplied the constant –1 by the **6** that was in front of the x^2 term. Now **divide BOTH of the constants** in **(x – 3) (x + 2)** by **6**

$$\left(x - \frac{3}{6} \right)\left(x + \frac{2}{6} \right)$$

reduce each fraction

$$\left(x - \frac{1}{2} \right)\left(x + \frac{1}{3} \right)$$

Step 5: "glide" the denominator of each fraction (if there is one) to the front of the x term

$$\left(2x - \frac{1}{2} \right)\left(3x + \frac{1}{3} \right)$$

Answer: $(2x - 1)(3x + 1)$

Tool 7A

Factoring Hard Trinomials like $Ax^2 \pm Bx \pm C$ where $A > 1$

that END with a POSITIVE number (C is POSITIVE)

by

the AC – Factoring By Grouping Method

Example 21

Factor: $3x^2 + 7x + 2$

Step 1: Multiply the two outer terms

$3x^2$ and the **2** to get **$+ 6x^2$**

$3x^2 + 7x + 2$

Step 2: Find 2 terms that **multiply to** $+ 6x^2$ and **add to + 7x**

+ 6x and + 1x

Step 3: Replace the **+ 7x** in

$3x^2$ **+ 7x + 2** with

+ 6x + 1x

$3x^2$ ___ ___ + 2

to get

$3x^2$ **+ 6x + 1x + 2**

Step 4: **$3x^2$ + 6x + 1x + 2**

Factor a GCF of 3x out of the first two terms and
Factor a GCF of 1 out of the last two terms

3x (x + 2) + 1(x + 2)

Factor a GCF of (x + 2) out of the two terms

Answer: (x + 2) (3x + 1)

Tool 7B

Factoring Hard Trinomials like $Ax^2 \pm Bx \pm C$ where $A > 1$
that END with a NEGATIVE number (C is NEGATIVE)
by

the AC – Factoring By Grouping Method

Example 22

Factor: $12x^2 + 5x - 2$

Step 1: Multiply the two outer terms
$12x^2$ and the **– 2** to get **$-24x^2$**

Step 2: Find 2 terms that **multiply to $-24x^2$** and **subtract to + 5x**
+ 8x and – 3x

Replace the **+ 5x** in
$12x^2$ + 5x – 2 with

+ 8x – 3x

$12x^2$ ___ ___ – 2

to get
$12x^2 + 8x - 3x - 2$

Step 4: **$12x^2 + 8x - 3x - 2$**

Factor a GCF of 4x out of the first two terms and
Factor a GCF of – 1 out of the last two terms

4x (3x + 2) – 1 (3x + 2)

Factor a GCF of (3x + 2) out of the two terms

Answer: (3x + 2) (4x – 1)

