

Section 4 – 6B:

Factoring Trinomials of the Form

$$1x^2 \pm Bx \pm C$$

The FOIL process changes a product of 2 binomials into a polynomial. The reverse process starts with a polynomial and finds the 2 binomials whose product will be that polynomial. The process of changing a polynomial into **a product of 2 binomials** is called **factoring a polynomial**.

This unit will look at factoring two types of trinomials.

A second degree **Trinomial**
that has $1x^2$ as the first term and
ends with a positive number

C is positive

$$1x^2 \pm Bx + C$$

A second degree **Trinomial**
that has $1x^2$ as the first term and
ends with a negative number

C is negative

$$1x^2 \pm Bx - C$$

The technique for factoring each type is similar. We will start with the type where **C is positive**.

How to factor $x^2 \pm Bx + C$ If the Trinomial **Ends in a Positive Number**

If the last term of $x^2 \pm Bx + C$ **is positive**

then we need two numbers **D** and **E** that

MULTIPLY to give **+ C**
and **ADD** to give **B**

and the factors are written as

$$(x \pm D)(x \pm E)$$

Example 1

Factor $x^2 - 9x + 20$
into $(x \pm D)(x \pm E)$

$$1 \cdot 20$$

$$2 \cdot 10$$

$$4 \cdot 5$$

$$x^2 - 9x + 20$$

we need two numbers

D and **E** that
multiply to + 20
and **add to - 9**

- 4 and **- 5** work

Answer: $(x - 4)(x - 5)$

Math 120 Section 4 – 6A

Example 2

Factor $x^2 + 9x + 18$
into $(x \pm D)(x \pm E)$

$$1 \cdot 18$$

$$2 \cdot 9$$

$$3 \cdot 6$$

$$x^2 + 9x + 18$$

we need two numbers

D and **E** that
multiply to + 18
and **add to + 9**

+ 3 and **+ 6** work

Answer: $(x + 3)(x + 6)$

Page 1

Example 3

Factor $x^2 - 7x + 12$
into $(x \pm D)(x \pm E)$

$$1 \cdot 12$$

$$2 \cdot 6$$

$$3 \cdot 4$$

$$x^2 - 7x + 12$$

we need two numbers

D and **E** that
multiply to + 12
and **add to - 7**

- 3 and **- 4** work

Answer: $(x - 3)(x - 4)$

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Example 4

Factor $x^2 + 12x + 20$
into $(x \pm D)(x \pm E)$

$1 \cdot 20$

$2 \cdot 10$

$4 \cdot 5$

$x^2 + 12x + 20$

we need two numbers

D and **E** that
multiply to + 20
and **add to + 12**

+ 2 and **+ 10** work

Answer: $(x + 2)(x + 10)$

Example 5

Factor $x^2 - 19x + 18$
into $(x \pm D)(x \pm E)$

$1 \cdot 18$

$2 \cdot 9$

$3 \cdot 6$

$x^2 - 19x + 18$

we need two numbers

D and **E** that
multiply to + 18
and **add to - 19**

- 1 and **- 18** work

Answer: $(x - 1)(x - 18)$

Example 6

Factor $x^2 + 8x + 12$
into $(x \pm D)(x \pm E)$

$1 \cdot 12$

$2 \cdot 6$

$3 \cdot 4$

$x^2 + 8x + 12$

we need two numbers

D and **E** that
multiply to + 12
and **add to + 8**

+ 2 and **+ 6** work

Answer: $(x + 2)(x + 6)$

Note: The order of the factors does not matter. Either $(x + 2)(x + 10)$ or $(x + 10)(x + 2)$ is correct.

Note: If none of the products of the last term work then the trinomial does not factor (**DNF or prime**)

Example 7

Factor $x^2 - 21x + 20$
into $(x \pm D)(x \pm E)$

$1 \cdot 20$

$2 \cdot 10$

$4 \cdot 5$

$x^2 - 21x + 20$

we need two numbers

D and **E** that
multiply to + 20
and **add to - 21**

- 1 and **- 20** work

$(x - 1)(x - 20)$

Answer: $(x - 1)(x - 20)$

Example 8

Factor $x^2 + 11x + 18$
into $(x \pm D)(x \pm E)$

$1 \cdot 18$

$2 \cdot 9$

$3 \cdot 6$

$x^2 + 11x + 18$

we need two numbers

D and **E** that
multiply to + 18
and **add to + 11**

+ 2 and **+ 9** work

$(x + 2)(x + 9)$

Answer: $(x + 2)(x + 9)$

Example 9

Factor $x^2 - 13x + 12$
into $(x \pm D)(x \pm E)$

$1 \cdot 12$

$2 \cdot 6$

$3 \cdot 4$

$x^2 - 13x + 12$

we need two numbers

D and **E** that
multiply to + 12
and **add to - 13**

- 1 and **- 12** work

$(x - 1)(x - 12)$

Answer: $(x - 1)(x - 12)$

Note: The order of the factors does not matter. Either $(x - 1)(x - 20)$ or $(x - 20)(x - 1)$ is correct.

Note: If none of the products of the last term work then the trinomial does not factor (**DNF or prime**)

How to factor $x^2 \pm Bx - C$ If the Trinomial **Ends in a Negative Number**

If the last term of $x^2 \pm Bx - C$ **is positive**

then we need two numbers **D** and **E** that

MULTIPLY to give **+ C**
and **SUBTRACT** to give **B**

and the factors are written as

$$(x \pm D)(x \pm E)$$

Example 1

Factor $x^2 - x - 20$
into $(x \pm D)(x \pm E)$

$$1 \cdot 20$$

$$2 \cdot 10$$

$$4 \cdot 5$$

$$x^2 - x - 20$$

we need two numbers

D and **E** that

multiply to - 20

and **subtract to - 1**

- 5 and **4** work

Answer: $(x - 5)(x + 4)$

Example 2

Factor $x^2 + 3x - 18$
into $(x \pm D)(x \pm E)$

$$1 \cdot 18$$

$$2 \cdot 9$$

$$3 \cdot 6$$

$$x^2 + 3x - 18$$

we need two numbers

D and **E** that

multiply to - 18

and **subtract to + 3**

+ 6 and **- 3** work

Answer: $(x + 6)(x - 3)$

Example 3

Factor $x^2 - 4x - 12$
into $(x \pm D)(x \pm E)$

$$1 \cdot 12$$

$$2 \cdot 6$$

$$3 \cdot 4$$

$$x^2 - 4x - 12$$

we need two numbers

D and **E** that

multiply to - 12

and **subtract to - 4**

- 6 and **+ 2** work

Answer: $(x - 6)(x + 2)$

Note: The order of the factors does not matter. Either $(x - 5)(x + 4)$ or $(x + 4)(x - 5)$ is correct.

Note: If none of the products of the last term work then the trinomial does not factor (**DNF or prime**)

Example 4

Factor $x^2 - 3x - 28$
into $(x \pm D)(x \pm E)$

$$1 \cdot 28$$

$$2 \cdot 14$$

$$4 \cdot 7$$

$$x^2 + 12x - 28$$

we need two numbers

D and **E** that

multiply to -28

and **subtract to -3**

$+4$ and -7 work

Answer: $(x+4)(x-7)$

Example 5

Factor $x^2 - 15x - 16$
into $(x \pm D)(x \pm E)$

$$1 \cdot 16$$

$$2 \cdot 8$$

$$4 \cdot 4$$

$$x^2 - 15x - 16$$

we need two numbers

D and **E** that

multiply to -16

and **subtract to -15**

-16 and $+1$ work

Answer: $(x-16)(x+1)$

Example 6

Factor $x^2 + 13x - 30$
into $(x \pm D)(x \pm E)$

$$1 \cdot 30$$

$$5 \cdot 6$$

$$2 \cdot 15$$

$$x^2 + 13x - 30$$

we need two numbers

D and **E** that

multiply to -30

and **subtract to $+13$**

$+15$ and -2 work

Answer: $(x+15)(x-2)$

Example 7

Factor $x^2 - 3x - 40$
into $(x \pm D)(x \pm E)$

$$1 \cdot 40$$

$$2 \cdot 20$$

$$4 \cdot 10$$

$$5 \cdot 8$$

$$x^2 - 3x - 40$$

we need two numbers

D and **E** that

multiply to -40

and **subtract to -3**

-8 and $+5$ work

Answer: $(x-8)(x+5)$

Example 8

Factor $x^2 + 5x - 24$
into $(x \pm D)(x \pm E)$

$$1 \cdot 24$$

$$2 \cdot 12$$

$$3 \cdot 8$$

$$4 \cdot 6$$

$$x^2 + 5x - 24$$

we need two numbers

D and **E** that

multiply to -24

and **subtract to $+5$**

$+8$ and -3 work

Answer: $(x+8)(x-3)$

Example 9

Factor $x^2 - 8x - 48$
into $(x \pm D)(x \pm E)$

$$1 \cdot 48$$

$$2 \cdot 24$$

$$3 \cdot 16$$

$$4 \cdot 12$$

$$6 \cdot 8$$

$$x^2 - 8x - 48$$

we need two numbers

D and **E** that

multiply to -48

and **subtract to -8**

-12 and $+4$ work

Answer: $(x-12)(x+4)$

Note: The order of the factors does not matter. Either $(x-5)(x+4)$ or $(x+4)(x-5)$ is correct.

Note: If none of the products of the last term work then the trinomial does not factor (**DNF or prime**)

Factoring Completely: Putting It all Together

Many Polynomials have a GCF that can be factored out and the expression left inside the parenthesis can then also be factored. This requires **two steps to completely factor the initial polynomial**.

Step 1: The first step in any factoring process is to **factor out the GCF** if there is one:

Example 1

$$\begin{aligned} 27x^2 - 12 \\ = 3(9x^2 - 4) \end{aligned}$$

Example 2

$$\begin{aligned} 2x^2 - 8x - 10 \\ = 2(x^2 - 4x - 5) \end{aligned}$$

Example 3

$$\begin{aligned} 4x^2 + 16x + 12 \\ = 4(x^2 + 4x + 3) \end{aligned}$$

At this point you will have either a **Binomial** or a **Trinomial inside the parenthesis**.

Step 2: Factor the expression inside the parenthesis if possible:

If there is a Binomial inside the parenthesis:

1. If it is the **Difference of Two Perfect Squares** then factor using $(a^2 - b^2) = (a + b)(a - b)$.

Example 4

$$\begin{aligned} 2(x^2 - 9) \\ = 2(x + 3)(x - 3) \end{aligned}$$

Example 5

$$\begin{aligned} 3x(4x^2 - 1) \\ = 3x(2x - 1)(2x + 1) \end{aligned}$$

2. If it is the **Sum of Two Perfect Squares** it cannot be factored. **Factoring is complete.**

$$2(x^2 + 9) \text{ Factoring is complete}$$

$$3x^2(4x^2 + 25) \text{ Factoring is complete}$$

If there is a Trinomial inside the parenthesis:

1. If the sign of the **last term of the trinomial is positive** look for the factors of the last term that **multiply to give the last term** and **add to give the middle term** of the trinomial.
2. If the sign of the **last term of the trinomial is negative** look for the factors of the last term that **multiply to give the last term** and **subtract to give the middle term** of the trinomial.

Example 6

$$\begin{aligned} 2x(x^2 - x + 12) \\ = 2x(x - 4)(x + 3) \end{aligned}$$

Example 7

$$\begin{aligned} 7x(x^2 + 3x - 10) \\ = 7x(x + 5)(x - 2) \end{aligned}$$

If the trinomial inside the parenthesis is not any of the types listed above we will not discuss how it may factor until a later course. For the purposes of this course **factoring is complete**.

Factoring a Polynomial Completely

To factor any polynomial completely you should **factor the GCF out FIRST** if the polynomial has a GCF. If the polynomial remaining inside the parentheses is a trinomial that can be factored then the complete factored answer is the product of the GCF and the factored trinomial.

1. Factor out the **GCF** if there is one
2. Factor the polynomial inside the parentheses If it can be factored.
3. The final answer is **the product of the GCF and the other factors.**

Example 1

$$3x^2 - 9x - 12 \text{ (the GCF is 3)}$$

$$= 3(x^2 - 3x - 4) \text{ factor}$$

$$= 3(x - 4)(x + 1)$$

The answer can be
written either

$$3(x - 4)(x + 1)$$

or

$$3(x + 1)(x - 4)$$

Example 2

$$2x^2 - 18x + 36 \text{ (the GCF is 2)}$$

$$= 2(x^2 - 9x + 18) \text{ (factor)}$$

$$= 2(x - 6)(x - 3)$$

The answer can be
written either

$$2(x - 6)(x - 3)$$

or

$$2(x - 3)(x - 6)$$

Example 3

$$4x^2 - 4x - 80 \text{ (the GCF is 4)}$$

$$= 4(x^2 - x - 20) \text{ (factor)}$$

$$= 4(x - 5)(x + 4)$$

The answer can be
written either

$$4(x - 5)(x + 4)$$

or

$$4(x + 4)(x - 5)$$

Example 4

$$5x^2 - 30x + 45 \text{ (the GCF is 5)}$$

$$= 5(x^2 - 6x + 9) \text{ (factor)}$$

$$= 5(x - 3)(x - 3)$$

Example 5

$$5x^2 - 10x + 20 \text{ (the GCF is 5)}$$

$$= 5(x^2 - 2x + 4)$$

Can't be factored further

$$= 5(x^2 - 2x + 4)$$

Example 6

$$x^2 - 7x + 5$$

There is no GCF and

the trinomial does not factor

so this **DOES NOT FACTOR**

To understand how to factor $1x^2 - Bx - C$ into $(x + D)(x - E)$

we need to start with multiplying

$(x + D)(x - E)$ by the FOIL process

to see how it is related to the trinomial $1x^2 - Bx - C$

$$\begin{array}{cccc} & F & O & I & L \\ (x + D)(x - E) & = & x \bullet x & + & Dx & + & Ex & + & (D)(-E) \\ & & \downarrow & & \underbrace{Dx - Ex} & & \downarrow & & \\ & & & & = & x^2 & + & (D - E)x & - & D \bullet E \\ (x + D)(x - E) & = & x^2 & + & (D - E)x & - & D \bullet E \\ & & \text{first} & & \text{middle} & & \text{last} \end{array}$$

If $x^2 - Bx - C$ must equal $(x + D)(x - E)$

then

$$\begin{aligned} & x^2 - Bx - C \\ = & x^2 + (D - E)x - DE \end{aligned}$$

and D and E must

multiply to give - C

and

subtract to give - B

If D and E must multiply to give - C (a negative number) then one of the numbers D or E must be positive and the other negative

If D and E must subtract to give the middle term Bx then

If the middle term is negative then D - E must be negative

If the middle term is Positive then D - E must be positive