Section 4 – 5B: Factoring Perfect Cubed Binomials

What is a Perfect Cube?

The product of three of the same factors will produce a a **Perfect Cube**.

$1 \bullet 1 \bullet 1 = \boxed{1}$ $\quad 2 \bullet 2 \bullet 2 = \boxed{8}$ $\quad 3 \bullet 3 \bullet 3 = \boxed{27}$ $\quad 4 \bullet 4 \bullet 4 = \boxed{64}$

$5 \bullet 5 \bullet 5 = \boxed{125}$ $\quad x \bullet x \bullet x = \boxed{x^3}$ $\quad x^2 \bullet x^2 \bullet x^2 = \boxed{x^6}$ $\quad x^3 \bullet x^3 \bullet x^3 = \boxed{x^9}$

The Difference of Two Perfect Cubes

A special **binomial** (2 terms) called **The Difference of Two Perfect Cubes** can be formed by taking any two perfect cubed terms and **subtracting** them.

$\boxed{64}\boxed{x^3} - \boxed{1}$ $\quad \boxed{8}\boxed{x^3} - \boxed{27}$ $\quad \boxed{64}\boxed{x^6} - \boxed{125}$

The Sum of Two Perfect Cubes

A special **binomial** (2 terms) called **The Sum of Two Perfect Cubes** can be formed by taking any two perfect cubed terms and **adding** them.

$\boxed{27}\boxed{x^3} + \boxed{64}$ $\quad \boxed{125}\boxed{x^3} + \boxed{8}$ $\quad \boxed{27}\boxed{x^6} + \boxed{1}$

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Factoring The Difference of Two Perfect Cubes

Factor: $a^3 - b^3$

$= (a \;-\; b) \quad (a^2 + \;\; ab \;\; + \;\; b^2)$

$(a-b)$ is found by $\begin{pmatrix} \text{perfect cube root} & \text{keep the} & \text{perfect cube root} \\ \text{of the first term} & \text{same sign} & \text{of the last term} \end{pmatrix}$

$(a \;-\; b) \quad (a^2 + \;\; ab \;\; + \;\; b^2)$ is found by

$(\text{First} - \text{Last}) \; (\text{First}^2 \;\; \text{change sign} \;\; \text{First} \bullet \text{Last} \;\; \text{always} + \;\; \text{Last}^2)$

Factoring The Difference of Two Perfect Cubes

Factor: $a^3 - \; b^3 =$

$\downarrow$

$= (a \;-\; b) \; (a^2 \;+\;\; ab \;\;+\;\; b^2)$

$= (\text{F} - \;\text{L}) \; (\text{F}^2 \;\; \text{cs to } + \;\; \text{F} \bullet \text{L} \; + \; \text{L}^2)$

cs means change the sign that was $-$ in $(\text{F} - \; \text{L})$
to $+$ for the middle term of the trinomial

the last sign in front of L^2 is always positive

Example 1

Factor

$x^3 - 27 =$

$(x-3)(x^2 + 3x + 9)$

Example 2

Factor

$8x^3 - 125 =$

$(2x-5)(4x^2 + 10x + 25)$

Example 3

Factor

$64x^3 - 27 =$

$(4x-3)(4x^2 + 12x + 9)$

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Factoring The Sum of Two Perfect Cubes

Factor: $a^3 + b^3$

$= (a + b) \quad (a^2 - ab + b^2)$

$(a+b)$ is found by $\begin{pmatrix} \text{perfect cube root} & \text{keep the} & \text{perfect cube root} \\ \text{of the first term} & \text{same sign} & \text{of the last term} \end{pmatrix}$

$(a + b) \quad (a^2 - ab + b^2)$ is found by

$(\text{First} - \text{Last}) \left(\text{First}^2 \ \text{change sign} \ \text{First} \bullet \text{Last} \ \ \text{always} + \ \text{Last}^2\right)$

Factoring The Sum of Two Perfect Cubes

Factor: $a^3 + b^3$

$\downarrow$

$= (a + b)\,(a^2 - ab + b^2)$

$= (\text{F} + \text{L})\,(\text{F}^2 \ \text{cs to} - \ \text{F} \bullet \text{L} + \text{L}^2)$

cs means change the sign that was $+$ in $(\text{F} + \text{L})$
to $-$ for the middle term of the trinominal

the last sign in front of L^2 is always positive

Example 4

Factor

$x^3 + 125 =$

$(x+5)(x^2 - 5x + 25)$

Example 5

Factor

$27x^3 - 8 =$

$(3x+2)(9x^2 - 6x + 4)$

Example 6

Factor

$125x^3 + 64 =$

$(5x+4)(25x^2 - 20x + 16)$

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Higher Power of x Examples

Example 7

Factor

$x^6 + 64 =$

$\left(x^2 + 4\right)\left(x^4 - 4x^2 + 16\right)$

Example 8

Factor

$x^8 + 125 =$

$\left(x^4 + 5\right)\left(x^8 - 5x^4 + 25\right)$

Factor Completely

Remember to factor out the GCF first if there is one.

Example 9

Factor

$24x^3 - 3$ (the GCF is 3)

$= 3(8x^3 - 1)$ Diff. of Cubes

$= 3(x - 1)(x^2 + x + 1)$

Example 10

Factor

$54x^2 + 16$ (the GCF is 2)

$= 2(27x^2 + 8)$ Sum of Cubes

$= 2(3x + 2)(9x^2 - 6x + 4)$

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