Section 4 – 4: Greatest Common Factor (GCF)

Factoring

The last chapter introduced the distributive process. The distributive process takes a product of a monomial and a polynomial and **changes the multiplication problem into a polynomial** (the addition or subtraction of several terms).

The distributive process changes
the product $2(3x+5)$
into the polynominal
$6x+10$

The distributive process changes
the product $4x(2x+3)$
into the polynominal
$8x^2+12x$

The last chapter also introduced the **FOIL** process. The **FOIL** process is a multiplication problem. The FOIL process takes the product of two binomials and **changes the multiplication problem into a polynomial.**

The FOIL process changes
the product $(x+1)(x+5)$
into the polynominal
x^2+6x+5

The FOIL process changes
the product $(x+2)(x-2)$
into the polynominal
x^2-4

Factoring

Factoring is the process of changing a polynomial into either a distributive problem or a FOIL problem. **Factoring reverses the distributive or FOIL process.**

If the distributive process changes
$2(3x+5)$ into $6x+10$
then the factoring process reverses this
and changes
$6x+10$ into $2(3x+5)$

If the FOIL process changes
$(x-2)(x+5)$ into $x^2+3x-10$
then the factoring process reverses this
and changes
$x^2+3x-10$ into $(x-2)(x+5)$

There are three types of factoring that will be covered in this chapter.

1. Factoring out the **Greatest Common Factor** (GCF).
2. Factoring the **Difference Of Two Perfect Squares.**
3. **Factoring Trinomials.**

Factors

The **factors of a whole number** are all the whole numbers that **divide evenly** into that number.

The factors of 4 are	The factors of 12 are	The factors of 5 are
1, 2 and 4	1, 2, 3, 4, 6 and 12	1 and 5

Greatest Common Factor (GCF)

The Greatest Common Factor (**GCF**) for a group of whole numbers is
the largest whole number
that will divide evenly into **every number in the group** of numbers.

Example 1
6
is the largest number
that will divide into
12 and 30
so the GCF
is 6

Example 2
12
is the largest number
that will divide into
12 and 24
so the GCF
is 12

Example 3
1
is the largest number
that will divide into
3 and 7
so the GCF
is 1

Example 4
2
is the largest number
that will divide into
6 , 12 and 20
so the GCF
is 2

Example 5
5
is the largest number
that will divide into
5 , 10 and 30
so the GCF
is 5

Example 6
1
is the largest number
that will divide into
2 , 10 and 11
so the GCF
is 1

Problems: **Find the Greatest Common Factor** (GCF) for each group of numbers:

1. 8 and 20 ______
2. 6 and 15 ______
3. 6 and 12 ________
4. 3 and 10 ______
5. 8, 16 and 32 ______
6. 14, 21 and 28 ________
7 3, 9 and 18 ______
8. 5, 8 and 12 ______
9. 8, 12 and 28 ________

Answers:

1. GCF is 4 2. GCF is 3 3. GCF is 6 4. GCF is 1 5. GCF is 8
6. GCF is 7 7. GCF is 3 8. GCF is 1 9. GCF is 4

Greatest Common Factor for Variable Terms

The Greatest Common Factor (GCF) can also be found for a list of variable terms.

If the list of variable terms has **a common variable**
then the GCF is the common variable
to the **lowest power that the common variable has in the group of terms.**

Example 1

x^2 and x^6
have a common variable of x

x^2 is the lowest power of x
in the two terms so

x^2 is the GCF

Example 2

y^3 and y^4
have a common variable of y

y^3 is the lowest power of y
in the two terms so

y^3 is the GCF

Example 3

x and x^3
have a common variable of x

x is the lowest power of x
in the two terms so

x is the GCF

Example 4

$x^4, x^2,$ and x^5
have a common variable of x

x^2 is the lowest power of x
in the three terms so

x^2 is the GCF

Example 5

y^3, y^4 and y
have a common variable of y

y is the lowest power of y
in the three terms so

y is the GCF

Example 6

x^3y^3 , xy^2 and y^5
have a common variable of y

y^2 is the lowest power of y
in the three terms so

y^2 is the GCF

Problems : Find the Greatest Common Factor for each list of variables:

1. The GCF of x^2 and x^3 is ______
2. The GCF of x^4 and x^5 is _______
3. The GCF of y^3 and y^6 is ______
4. The GCF of y^4, y^3 and y^5 is _______
5. The GCF of x^2, x^3 and x^4 is ______
6. The GCF of x, x^2 and x^3 is ______
7. The GCF of x^4 , x^3y and xy is _______
8. The GCF of x^4 and y^4 is _______

Answers:

1. GCF is x^2
2. GCF is x^4
3. GCF is y^3
4. GCF is y^3
5. GCF is x^2
6. GCF is x
7. GCF is x
8. no common variable

Finding the Greatest Common Factor (GCF) for a Polynomial

Each term in a polynomial may have both a number coefficient and one or more variables. To find the GCF for a polynomial **find the GCF for all the coefficients** and then fi**nd the GCF for the variables in each term** and then **multiply the GCF's together.**

Finding the Greatest Common Factor

1. Find the GCF for the coefficients (number in front a variable) of all the terms.
2. Find the GCF for each variable term by using the **lowest power** that it has in the group of terms.

3, The GCF is the product of steps 1 and 2.

Example 1:

Find the GCF of $8x^2 + 12x$

1. The GCF of 8 and 12 is **4**
2. The GCF of x^2 and x is x

The GCF is $4x$

Example 2:

Find the GCF of $10y^4 + 15y^3$

1. The GCF of 10 and 15 is **5**
2. The GCF of y^4 and y^3 is y^3

The GCF is $5y^3$

Example 3:

Find the GCF of $12x^4 + 9x^3 + 6x^2$

1. The GCF of 12 , 9, and 6 is **3**
2. The GCF of x^4 , x^3, x^2 is x^2

The GCF is $3x^2$

Example 4:

Find the GCF of $4y^5 + 8y^4 + 10y^3$

1. The GCF of 4 , 8, and 10 is **2**
2. The GCF of y^5 , y^4 , y^3 is y^3

The GCF is $2y^3$

Example 5:

Find the GCF of $3y^5 + 8y^4 + 12y^3$

1. The GCF of 3 , 8, and 12 is 1
2. The GCF of y^5 , y^4 , y^3 is y^3

The GCF is y^3

Example 6:

Find the GCF of $16x^3 + 12x^2 + 8$

1. The GCF of 16 , 12, and 8 is 4
2. There is no common variable so the GCF is 1

The GCF is 4

Factoring out the GCF from a Polynomial

Factoring out a GCF from a polynomial is the reverse process of the distributive operation. The distributive operation **multiplied** a term outside a parentheses into each term of an expression inside the parentheses as seen in the following examples:

$2(5x+3)=10x+6$ $\quad$ $5x(2x-4)=10x^2-20x$ $\quad$ $3x(2x^2-5x)=6x^3-15x^2$

Factoring out the GCF involves DIVIDING OUT THE GCF and placing it outside the parentheses as a product with the results of the division left inside the parentheses.

Example 1. Factor the GCF out of $8x+16$

$8x+16$ has a GCF of 8. Factoring (dividing) a 8 out of $8x+16$ results in $8\left(\frac{8x}{8}+\frac{16}{8}\right)$

When we reduce each fraction we get $8(x+2)$

The factored form of $8x+16$ is $8(x+2)$

Check: You can check your answer by distributing the GCF back in.

Example 2. Factor the GCF out of $15x^2-10x$

$15x^2-10x$ has a GCF of $5x$. Factoring (dividing) a $5x$ out of $15x^2-10x$ results in $5x\left(\frac{15x^2}{5x}-\frac{10x}{5x}\right)$

When we reduce each fraction we get $5x(3x-2)$

The factored form of $15x^2-10x$ is $5x(3x-2)$

Example 3

Factor

$6y^2-10y$ (GCF is $2y$)

$2y\left(\frac{6y^2}{2y}-\frac{10y}{2y}\right)$

$=2y(3y-5)$

Example 4

Factor

$12x^3+4x^2$ (GCF is $4x^2$)

$4x^2\left(\frac{12x^3}{4x^2}+\frac{4x^2}{4x^2}\right)$

$=4x^2(3x+1)$

Example 5

Factor

$12y^4+27y^3-6y^2$ (GCF is $3y^2$)

$3y^2\left(\frac{12y^4}{3y^2}+\frac{27y^3}{3y^2}-\frac{6y^2}{3y^2}\right)$

$=3y^2(4y^2+9y-2)$

Factoring Out the GCF

Example 6

Factor: $12x^2 - 10x + 6$

(the GCF is 2)

$= 2(6x^2 - 5x + 3)$

Example 7

Factor: $5x^2y + 20xy - 5y$

(the GCF is 5y)

$= 5y(x^2 + 4x - 1)$

Example 8

Factor: $10x^3 - 20x^2 + 50x$

(the GCF is $10x$)

$= 10x(x^2 - 2x + 5)$

