Section 4 – 3: Synthetic Division

Use Synthetic Division to do the following problems:

Example 1

$$\frac{3x^3 + x^2 - 4x - 13}{x - 2} =$$

$$\begin{array}{r|cccc} 2 & 3 & 1 & -4 & -13 \\ & \downarrow & 6 & 14 & 20 \\ \hline & 3 & 7 & 10 & 7 \end{array}$$

$$= 3x^2 + 7x + 10 + \frac{7}{x-2}$$

Example 2

$$\frac{x^3 + 2x^2 - 5x - 10}{x + 3} =$$

$$\begin{array}{r|cccc} -3 & 1 & +2 & -5 & -10 \\ & \downarrow & -3 & 3 & 6 \\ \hline & 1 & -1 & -2 & -4 \end{array}$$

$$= x^2 - x - 2 - \frac{4}{x+3}$$

Example 3

$$\frac{x^3 + 6x^2 + 5x - 3}{x + 4} =$$

$$\begin{array}{r|cccc} -4 & 1 & 6 & 5 & -3 \\ & \downarrow & -4 & -8 & 12 \\ \hline & 1 & 2 & -3 & 9 \end{array}$$

$$= x^2 + 2x - 3 + \frac{9}{x+4}$$

Example 4

$$\frac{x^4 + 5x^3 + 8x^2 + 5x - 4}{x + 3} =$$

$$\begin{array}{r|ccccc} -3 & 1 & 5 & 8 & 5 & 4 \\ & \downarrow & -3 & -6 & -6 & 3 \\ \hline & 1 & 2 & 2 & -1 & 7 \end{array}$$

$$= x^3 + 2x^2 + 2x - 1 + \frac{7}{x+3}$$

Example 5

$$\frac{2x^4 - 9x^3 - 5x^2 + 3x - 15}{x - 5} =$$

$$\begin{array}{r|ccccc} 5 & 2 & -9 & -5 & 3 & -15 \\ & \downarrow & 10 & 5 & 0 & 15 \\ \hline & 2 & 1 & 0 & 3 & 0 \end{array}$$

$$= 2x^3 + x^2 + 3$$

Example 6

$$\frac{4x^3 - 2x - 1}{x + 1} =$$

$$\begin{array}{r|cccc} -1 & 4 & 0 & -2 & -1 \\ & \downarrow & -4 & 4 & -2 \\ \hline & 4 & -4 & 2 & -3 \end{array}$$

$$= 4x^2 - 4x + 2 - \frac{3}{x+1}$$

Example 7

$$\frac{3x^3 + 17}{x + 2} =$$

$$\begin{array}{r|cccc} -2 & 3 & 0 & 0 & 17 \\ & \downarrow & -6 & 12 & -24 \\ \hline & 3 & -6 & 12 & -7 \end{array}$$

$$= 3x^2 - 6x + 12 - \frac{7}{x+2}$$

The Remainder Theorem

When a Polynomial P(x) is divided by $1x - a$ using long division

the reminder of the division is P(a)

Using Synthetic Division to find P(x)

To find P(x)

1. Use Synthetic Division to divide by x (no sign change).
2. The remainder of the Synthetic Division will be P(x)

Use the Synthetic Division to find P(c) for the given P(x)

Example 2

$P(x) = 4x^4 - 5x^3 - 2x^2 - 3x - 4$ find P(2)

$$\begin{array}{r|rrrrr} 2 & 4 & -5 & -2 & -3 & -4 \\ & \downarrow & 8 & 6 & 8 & 10 \\ \hline & 4 & 3 & 4 & 5 & 6 \end{array}$$

remainder is 5

so $P(2) = 6$

$$\begin{aligned} P(2) &= 4(2)^4 - 5(2)^3 - 2(2)^2 - 3(2) - 4 \\ &= 4(16) - 5(8) - 2(4) - 3(2) - 4 \\ &= 64 - 40 - 8 - 6 - 4 \\ &= 6 \end{aligned}$$

Example 2

$P(x) = 5x^4 + 9x^3 + 2x^2 + 3x + 7$ find P(−1)

$$\begin{array}{r|rrrrr} -1 & 5 & 9 & 2 & 3 & 7 \\ & & -5 & -4 & 2 & -5 \\ \hline & 5 & 4 & -2 & 5 & 2 \end{array}$$

remainder is 2

so $P(-1) = 2$

$$\begin{aligned} P(-1) &= 5(-1)^4 + 9(-1)^3 + 2(-1)^2 + 3(-1) + 7 \\ &= 5(1) + 9(-1) + 2(1) + 3(-1) + 7 \\ &= 5 - 9 + 2 - 3 + 7 \\ &= 2 \end{aligned}$$

