Section 4 – 2: Dividing Polynomials

Dividing Polynomials if the denominator is a monomial.

We add and subtract fractions with a common denominator using the following rule. If there is a common denominator then combine the numerators and put them over that common denominator.

$$\frac{A}{D}+\frac{B}{D}-\frac{C}{D}=\frac{A+B-C}{D}$$

This process could be written backwards and still be a true equation.

$$\frac{A+B-C}{D}=\frac{A}{D}+\frac{B}{D}-\frac{C}{D}$$

This rule states that if we have a polynomial with several terms in the numerator of a fraction then we can break that fraction into several separate fractions. Each of the separate fractions will have one term of the polynomial in the numerator and **the common denominator in the denominator.**

If the denominator is a monomial then each separate fraction can be reduced separately using the **Quotient Rule.** We are not really dividing the fraction. We are breaking it into several fractions each of which can be reduced separately using the **Quotient Rule.**

Why break up the fraction with a monomial in the denominator?

The quotient rule cannot be used if the numerator or denominator has more than one term. **We CANNOT cancel parts of the polynomial.**

The quotient rule can be used to reduce fractions
but t**he numerator and denominator must both be monomials.**

You cannot reduce the parts
of addition or subtraction
in the numerator
with the term in the denominator

$$\frac{5x^2+6}{3x}$$

the 3 and the 68
and the x and the x^2
cannot be reduced

$$\longrightarrow$$

but if you break up the fraction
by putting each term of the numerator
under a seperate common denominator

$$\frac{5x^2}{3x}+\frac{6}{3x}$$

the x and the x^2 in $\frac{5x^2}{3x}$

3 and the 6 in $\frac{6}{3x}$

can be reduced

If the **denominator of the fraction is a monomial** then you **break up each fraction** into its separate parts and then **reduce each separate fraction using the quotient rule.**

Example 1

$$\frac{8x^3 + 4x^2}{4x}$$

$$= \frac{8x^3}{4x} + \frac{4x^2}{4x}$$

$$= 2x^2 + x$$

Example 2

$$\frac{9xy - 4y}{12xy}$$

$$= \frac{9xy}{12xy} - \frac{4y}{12xy}$$

$$= \frac{3}{4} - \frac{1}{3x}$$

Example 3

$$\frac{8y^4 - 6y^3 + y^2}{4y^3}$$

$$= \frac{8y^4}{4y^3} - \frac{6y^3}{4y^3} + \frac{y^2}{4y^3}$$

$$= 2y - \frac{3}{2} + \frac{1}{4y}$$

Dividing Polynomials if the denominator is a Binomial: Long Division

Why breaking up the fraction with a binomial in the denominator will not work.

Breaking up a fraction into its separate parts and using the quotient rule to reduce each separate fraction will not work if the denominator is a binomial. If we did break up the fraction then each separate fraction would have the binomial as their denominator.

$$\frac{A+B-C}{A+B}=\frac{A}{A+B}+\frac{B}{A+B}-\frac{C}{A+B}$$

The quotient rule cannot be used to reduce each separate fraction if the numerator and denominator are not both monomials. We must develop another process to reduce the fraction. The process we use is to divide the binomial in the denominator into the numerator by using long division. This long division process will look like the long division process for fractions with whole numbers that you have used in the past. The process will be a bit more involved due to the nature of the algebraic terms involved.

Long division with whole numbers

$$\begin{array}{r} 4 \\ 2\overline{)97} \end{array}$$ divide the 2 outside into the first number inside (9) and put the results over the 9

multiply the 4 on top by the 2 outside and write the product 8 under the 9

$$\begin{array}{r} 4 \\ 2\overline{)97} \\ \underline{8} \end{array}$$ multiply the 8 in the new row by -1 to create a subtraction problem

$$\begin{array}{r} 4 \\ 2\overline{)97} \\ \underline{-8} \\ 17 \end{array}$$ combine the terms above the underline and bring down the 7

Start the process over again

$$\begin{array}{r} 48 \\ 2\overline{)\;97} \\ \underline{-8} \\ 17 \\ \underline{-16} \\ 1 \end{array}$$

17 Divide 2 into 17 to get 8. Put the 8 above the 7 and multiply the 2 and 8. Put the 16 under the 17

-16 multiply the 16 in the new row by -1 to create a subtraction problem

1 We are done and have a remainder of 1

the answer is $48\frac{1}{2}$

Example 1

$$\begin{array}{rl} & x \\ \cline{2-2} x-2 \big) & x^2-5x+1 \end{array}$$

divide the first term of the binomial outside into the first term inside $\frac{x^2}{x}=x$ and put the results above the division bar

multiply the x on top by the $x-2$ outside and write the product of $x(x-2)$ under the trinominal

$$\begin{array}{rl} & x \\ \cline{2-2} x-2 \big) & x^2-5x+1 \\ & \underline{x^2-2x} \end{array}$$

multiply EACH term in the new row by -1 to create a subtraction problem

$$\begin{array}{rl} & x \\ \cline{2-2} x-2 \big) & x^2-5x\ +1 \\ & \underline{-x^2+2x} \\ & \quad -3x+1 \end{array}$$

<u>combine the terms above the underline and bring down the $+1$</u>

Start the process over again with the $-3x+1$ term and the $x-2$ outside

$$\begin{array}{rl} & x-3 \\ \cline{2-2} x-2 \big) & x^2-5x+1 \\ & \underline{-x^2+2x} \\ & \quad -3x+1 \end{array}$$

divide the first term of the binomial outside into the first term of $-3x+1$ $\quad \frac{-3x}{x}=-3$

multiply the -3 on top by the $x-2$ outside $-3(x-2)$ and write the product under the $-3x+1$

$$\begin{array}{rl} & x-3 \\ \cline{2-2} x-2 \big) & x^2-5x+1 \\ & \underline{-x^2+2x} \\ & \quad -3x+1 \\ & \underline{\quad -3x+6} \end{array}$$

multiply EACH term in the new row by -1 to create a subtraction problem

$$\begin{array}{rl} & x-3 \\ \cline{2-2} x-2 \big) & x^2-5x+1 \\ & \underline{-x^2+2x} \\ & \quad -3x+1 \\ & \quad \underline{3x-6} \\ & \qquad -5 \end{array}$$

<u>combine the terms above the underline</u>

we are done and have a remainer of -5

the answer is $x-3-\frac{5}{x-2}$

Example 2

$$\begin{array}{r} x \\ x+3\overline{\big)x^2+5x+10} \end{array}$$

divide the first term of the binomial outside into the first term inside $\frac{x^2}{x} = x$ and put the results above the division bar

multiply the x on top by the $x+3$ outside and write the product of $x(x+3)$ under the trinominal

$$\begin{array}{r} x \\ x+3\overline{\big)x^2+5x+10} \\ \underline{x^2+3x} \end{array}$$

multiply EACH term in the new row by -1 to create a subtraction problem

$$\begin{array}{r} x \\ x+3\overline{\big)x^2+5x+10} \\ \underline{-x^2-3x} \\ 2x+10 \end{array}$$

combine the terms above the underline amd bring down the $+10$

Start the process over again with the $2x+10$ term and the $x+3$ outside

$$\begin{array}{r} x+2 \\ x+3\overline{\big)x^2+5x+10} \\ \underline{-x^2-3x} \\ 2x+10 \end{array}$$

divide the first term of the binomial outside into the first term of $2x+10$ $\quad \frac{2x}{x} = 2$

multiply the $+2$ on top by the $x-3$ outside $+2(x+3)$ and write the product under the $2x+10$

$$\begin{array}{r} x+2 \\ x+3\overline{\big)x^2+5x+10} \\ \underline{-x^2-3x} \\ 2x+10 \\ \underline{2x+6} \end{array}$$

multiply EACH term in the new row by -1 to create a subtraction problem

$$\begin{array}{r} x+2 \\ x+3\overline{\big)x^2+5x+10} \\ \underline{-x^2-3x} \\ 2x+10 \\ \underline{-2x-6} \\ +4 \end{array}$$

combine the terms above the underline

we are done and have a remainer of $+4$

the answer is $x+2+\frac{4}{x+3}$

Example 3 using a 0x as a place holder for a missing term

$$\begin{array}{r} x \\ x+4\overline{\big)x^2+0x+5} \end{array}$$

divide the first term of the binomial outside into the first term inside $\dfrac{x^2}{x}=x$ and put the results above the division bar

multiply the x on top by the $x+4$ outside and write the product of $x(x+4)$ under the trinominal

$$\begin{array}{r} x \\ x+4\overline{\big)x^2+0x+5} \\ \underline{x^2+4x} \end{array}$$

multiply EACH term in the new row by -1 to create a subtraction problem

$$\begin{array}{r} x \\ x+4\overline{\big)\;x^2+0x\;+5} \\ \underline{-x^2-4x} \\ -4x+5 \end{array}$$

<u>combine the terms above the underline and bring down the $+5$</u>

Start the process over again with the $-4x+5$ term and the $x+4$ outside

$$\begin{array}{r} x-4 \\ x+4\overline{\big)\;x^2+0x+5} \\ \underline{-x^2-4x} \\ -4x+5 \end{array}$$

divide the first term of the binomial outside into the first term of $-4x+5$ $\quad\dfrac{-4x}{x}=-4$

multiply the -4 on top by the $x+4$ outside $-4(x+4)$ and write the product under the $-4x+5$

$$\begin{array}{r} x-4 \\ x+4\overline{\big)\;x^2+0x+5} \\ \underline{-x^2-4x} \\ -4x+5 \\ \underline{-4x-16} \end{array}$$

multiply EACH term in the new row by -1 to create a subtraction problem

$$\begin{array}{r} x-4 \\ x+4\overline{\big)x^2+0x+5} \\ \underline{-x^2-4x} \\ -4x+5 \\ \underline{4x+16} \\ 21 \end{array}$$

<u>combine the terms above the underline</u>

21 we are done and have a remainer of 21

the answer is $x-4+\dfrac{21}{x+4}$

Example 4

$$\begin{array}{rl} & 3x \\ 2x+3 \big) \hspace{-4pt} & \overline{6x^2+15x-5} \end{array}$$

divide the first term of the binomial outside into the first term inside $\frac{6x^2}{2x} = 3x$
and put the results above the division bar

multiply the x on top by the $2x+3$ outside and write the product of $3x(2x+3)$ under the trinomial

$$\begin{array}{rl} & 3x \\ 2x+3 \big) \hspace{-4pt} & \overline{6x^2+15x-5} \\ & \underline{6x^2+9x} \end{array}$$

multiply EACH term in the new row by -1 to create a subtraction problem

$$\begin{array}{rl} & x \\ 2x+3 \big) \hspace{-4pt} & \overline{6x^2+15x-5} \\ & \underline{-6x^2-9x} \\ & \qquad 6x-5 \end{array}$$

combine the terms above the underline and bring down the -5

Start the process over again with the $6x-5$ term and the $2x+3$ outside

$$\begin{array}{rl} & x+3 \\ 2x+3 \big) \hspace{-4pt} & \overline{6x^2+15x-5} \\ & \underline{-6x^2-9x} \\ & \qquad 6x-5 \end{array}$$

divide the first term of the binomial outside into the first term of $6x-5$ $\frac{6x}{2x} = 3$

multiply the 3 on top by the $2x+3$ outside $3(2x+3)$ and write the product under the $6x-5$

$$\begin{array}{rl} & x\ +\ 3 \\ 2x+3 \big) \hspace{-4pt} & \overline{6x^2+15x-5} \\ & \underline{-6x^2-9x} \\ & \qquad 6x-5 \\ & \underline{\qquad 6x+9} \end{array}$$

multiply EACH term in the new row by -1 to create a subtraction problem

$$\begin{array}{rl} & x+3 \\ 2x+3 \big) \hspace{-4pt} & \overline{6x^2+15x-5} \\ & \underline{-6x^2-9x} \\ & \qquad 6x-5 \\ & \qquad \underline{-6x-9} \\ & \qquad -14 \end{array}$$

combine the terms above the underline

we are done and have a remainer of -14

the answer is $x+3-\frac{14}{2x+3}$