Section 4 – 1B: Multiplying Polynomials

The Distributive Property

The product of a monomial times a monomial was described as the Product Rule in Section 6-1. This section will further this concept by defining the product of a monomial times a binomial or a trinomial.

The Distributive Property

$$a(b+c)$$
$$= a \bullet b + a \bullet c$$

To distribute you **multiply the term outside the parentheses**

into **each term inside the parentheses.**

Example 1
A constant times a Binomial

$$-4(2x-5) \text{ means } -4(2x) \text{ and } -4(-5)$$
$$= -8x + 20$$

Example 2
A constant times a Binomial

$$\frac{3}{2}(6x-10) \text{ means } \frac{3}{2}(6x) \text{ and } \frac{3}{2}(-10)$$
$$= \frac{18}{2}x - \frac{30}{2} = 9x - 15$$

Example 3
A constant times a Trinomial

$$-2(3x^2 - 4x + 6) = -6x^2 + 8x - 12$$

Example 4
A constant times a Polynomial

$$2(5x^3 + 3x^2 - 1x - 4) = 10x^3 + 6x^2 - 2x - 8$$

We can also use the distributive property to **multiply a variable term and a polynomial.** You distribute the variable term into the parenthesis the same way you did the constant but you use the **product rule** for multiplying each variable term.

The Distributive Property

$$a(b+c)$$
$$= a \bullet b + a \bullet c$$

Product Rule : $a x^{n} \cdot b x^{m} = a \cdot b \cdot x^{n+m}$

Distributive Examples

Example1

$$4x(3x-4)$$
$$=12x^{2}-16x$$

Example 2

$$2x\left(3x^{2}+2x+3\right)$$
$$=6x^{3}+4x^{2}+6x$$

Example 3

$$-2x^{2}\left(x^{2}-3x-1\right)$$
$$=-2x^{4}+6x^{3}+2x^{2}$$

Example 4

$$-x^{3}\left(x^{2}+5x-12\right)$$
$$=-x^{5}-5x^{4}+12x^{3}$$

Example 5

$$\frac{4}{5}x^{2}\left(15x^{2}+5x-10\right)$$
$$=\frac{60}{5}x^{4}+\frac{20}{5}x^{3}-\frac{40}{5}x^{2}$$
$$=12x^{4}+4x^{3}-8x^{2}$$

Example 6

$$\frac{-2}{3}x\left(6x^{2}-9x-15\right)$$
$$=\frac{-12}{3}x^{3}+\frac{18}{3}x^{2}+\frac{30}{3}x$$
$$= \ =-4x^{3}+6x^{2}+10x$$

Distribute – Combine Like Terms

The Order of Operations rules require that multiplication be done before addition or subtraction. Since the distributive process is a multiplication step it must be done before the addition or subtraction of like terms. After the distributive process is complete, the remaining polynomial may be further simplified by combining any **Like Terms** that occur**.**

First **Distribute** the number outside the parentheses into the terms inside the parentheses and then **combine Like Terms.**

Example 1

$-2x(-2x+4)+10x$ (distribute)

$= 4x^2 - 8x + 10x$ (combine)

$= 4x^2 + 2x$

Example 2

$-9x^2 - (-5x^2 + 3x)$ (distribute)

$= -9x^2 + 5x^2 - 3x$ (combine)

$= -4x^2 - 3x$

Example 3

$2x - 5x(2x-3)$ (distribute)

$= 2x - 10x^2 + 15x$ (combine)

$= -10x^2 + 17x$

Example 4

$-3x(4x-3) + 2x^2$ (distribute)

$= -12x^2 + 9x + 2x^2$ (combine)

$= -10x^2 + 9x$

Example 5

$-(3x^2 + 5x - 9) - (2x - 3)$ (distribute)

$= -3x^2 - 5x + 9 - 2x + 3$ (combine)

$= -3x^2 - 7x + 12$

Example 6

$y(2y-3) - (6y-3)$ (distribute)

$= 2y^2 - 3y - 6y + 3$ (combine)

$= 2y^2 - 9y + 3$

FOIL Introduction

We have used the **Product Rule** to find the product of two monomials**:**

$$a x^n \cdot b x^m = a \cdot b \cdot x^{n+m}$$

$x \cdot x = x^2$ $\quad\quad$ $5x \cdot x^2 = 5x^3$ $\quad\quad$ $-2x^2 \cdot 3x^3 = -6x^5$

We have also used the **distributive rule** to find the product of a monomial and a polynomial

$x(x+1) = x^2 + x$ $\quad\quad$ $5(x+2) = 5x + 10$ $\quad\quad$ $3x(x+2) = 3x^2 + 6x$

We will now combine both of these rules to multiply **two binomials** like $(2x-3)(x+2)$

Each binomial has a first term and a last term. To multiply the two binomials we distribute the first term in the first binomial to both terms in the second binomial and then distribute the last term in the first binomial to both terms in the second binomial. **This will result in a polynomial with 4 terms**. The two middle terms are often like terms that can often be combined.

F. O. I. L.

We distribute both of the terms in the first binomial to both of the terms in the second binomial using the following order

We use the mnemonic **F O I L** to help us remember the process

F The product of the two **First** terms $(x-2)(x+3) = x^2$

O The product of the two **Outer** terms $(x-2)(x+3) = 3x$

I The product of the two **Inner** terms $(x-2)(x+3) = -2x$

L The product of the two **Last** terms $(x-2)(x+3) = -6$

$$(x-2)(x+3)$$

F O I L

$$x^2 + 3x - 2x - 6$$

$$= \mathbf{x^2 + x - 6}$$

Example 1

$(x-5)(x-2)=$

F O I L

$x^2-2x-5x+10$

$=\mathbf{x^2-7x+10}$

Example 2

$(2x-5)(3x+2)=$

F O I L

$6x^2+4x-15x-10$

$=\mathbf{6x^2-11x-10}$

Example 3

$(3x+2)(3x-2)=$

F O I L

$9x^2-6x+6x-4$

$=\mathbf{9x^2-4}$

Example 4

$(x-6)(x-2)$

F O I L

$x^2-2x-6x+12$

$=x^2-8x+12$

Example 5

$(x+5)(x-3)$

F O I L

$x^2-3x+5x-15$

$=x^2+2x-15$

Example 6

$(x+3)(x-3)$

F O I L

$x^2-3x+3x-9$

$=x^2-9$

Example 7

$(3x+2)(4x-1)$

F O I L

$12x^2-3x+8x-2$

$=12x^2+5x-2$

Example 8

$(x-3)^2$ means

$(x-3)(x-3)$

F O I L

$x^2-3x-3x+9$

$=x^2-6x+9$

Example 9

$(2x-5)^2$ means

$(2x-5)(2x-5)$

F O I L

$4x^2-10x-10x+25$

$=4x^2-20x+25$

We use the FOIL process to multiply a binomial times a binomial. We can extend this process to multiply a binomial times a trinomial. To multiply a trinomial by a binomial distribute the first term of the binomial into the trinomial and then distribute the second term of the binomial into the trinomial. After this step combine any like terms that exist.

Example 10

$$(2x-3)(x^2+5x+1)$$

distribute the $2x$

$$=2x^3+10x^2+2x$$

and then distribute the -3

$$-3x^2-15x-3$$

$$=2x^3+10x^2+2x-3x^2-15x-3$$

and then combine like terms

$$=2x^3+7x^2-13x-3$$

Example 11

$$(x+4)(x^2+2x-1)$$

distribute the x

$$x^3+2x^2-x$$

and then distribute the $+4$

$$+4x^2+8x-4$$

$$=x^3+2x^2-x+4x^2+8x-4$$

and then combine like terms

$$=x^3+6x^2+7x-4$$

Example 12

$$(3x-1)(3x^2+x+1)$$

distribute the $3x$

$$9x^3+3x^2+3x$$

and then distribute the -1

$$-3x^2-x-1$$

$$=9x^3+3x^2+3x-3x^2-x-1$$

and then combine like terms

$$=9x^3+2x-1$$

Example 13

$$(2x+3)(4x^2-6x+9)$$

distribute the $2x$

$$8x^3-12x^2+18x$$

and then distribute the $+3$

$$+12x^2-18x+27$$

$$=8x^3-12x^2+18x+12x^2-18x+27$$

and then combine like terms

$$=8x^3+27$$