Section 4 – 1A: Combining Like Terms in Polynomial

Polynomials

A **polynomial** is an expression that has two or more terms each separated by a + or – sign. If the expression has only one term it is called a **monomial.** If the expression has two terms it is called a **binomial.** If the expression has three terms it is called a **Trinomial.** The degree of the polynomial is determined by the highest power or exponent in any of the terms

a constnat is a zero degree monomial	a first degree monomial	a second degree binomial	a second degree binomial
5	$-2x$	$5x^2 - 25$	$3x^2 + 3x$

a second degree trinomial	a second degree trinomial	a third degree trinomial	a third degree trinomial
$-x^2 - 7x + 2$	$5x^2 + x - 1$	$3x^3 + 3x - 1$	$x^3 - 2x^2 - 4x$

Like Terms

A polynomial is an expression with several terms. **Like Terms** are terms that can be combined by addition or subtraction to form a single term**.** For terms to be considered like terms **every term** must have the **exact same variables** (letters) and each variable must have the **exact same exponent as all of the other terms**. The coefficients are not used to determine if the terms are alike.

Example of terms that are like terms include:

$5x^2$ and $3x^2$ are like terms because the x^2 in each term are the same letters to the exact same powers	$-4y^3$ and y^3 are like terms because the y^3 in each term are the same letters to the exact same powers	$2x^2y$ and $-5x^2y$ are like terms because the x^2y in each term are the same letters to the exact same powers

Example of terms that are NOT like terms include:

$5x^3$ and $3x^2$ are not like terms because the x^3 and x^2 are not to the same power	$-4x^2$ and y^2 are not like terms because the x and y terms are not the exact same variables	$2xy^2$ and $-5x^2y$ are not like terms because the xy^2 and x^2y terms have the same letters but they do not have the exact same exponents

Combining Like Terms

Combining like terms involves determining the **total of the coefficients** of all the like terms. If there are three **x** terms in an expression like $3x + 2x + 5x$ then you can combine all three of the **x** terms and express that total with only one **x** term. This is done by adding or subtracting the coefficients of the **x** terms and using that number as the coefficient of the variable term that was the like term.

Example 1

$$3x + 2x + 5x$$

To combine the like terms $3x + 2x + 5x$ you combine the $3 + 2 + 5$ to get 10 and use the common variable name **x** to get

$$3x + 2x + 5x$$
$$= 10x$$

Example 2

$$2x^2 + 4x^2 - 12x^2$$

To combine the like terms $2x^2 + 4x^2 - 12x^2$ you combine the $2 + 4 - 12$ to get –6 and use the common variable name x^2 to get

$$2x^2 + 4x^2 - 12x^2$$
$$= -6x^2$$

To Combine Like Terms

1. Combine the coefficients of the like terms by **adding or subtracting the coefficients.**

2. Using the number in step 1 as the coefficient of the variable term that was the like term.

Example 1

$$5x + 8x$$

combime the $5 + 8$

$$= 13x$$

Example 2

$$4y - 9y$$

combime the $4 - 9$

$$= -5y$$

Example 3

$$10xy - xy$$

combime the $10 - 1$

$$= 9xy$$

Example 5

$$-3x^2 - x^2$$
$$= -4x^2$$

Example 6

$$8x^2 - 12x^2$$

combime the $8 - 12$

$$= -4x^2$$

Example 7

$$3xy^2 + xy^2$$

combime the $3 + 1$

$$= 4xy^2$$

Example 9

$$2y^2 - 9y^2 + 3y^2$$

combime the $2 - 9 + 3$

$$= -4y^2$$

Example 10

$$3xy + 8xy - xy$$

combime the $3 + 8 - 1$

$$= 10xy$$

Example 11

$$2y - 7y + 5y$$

combime the $2 - 7 + 5$

$$= 0$$

Terms with Two Variables

List the variables in any single term in alphabetical order

If a **single** term has more then one variable we list the letters in **alphabetical order**. This allows us to more easily compare terms to see if they are alike. We do not write a term with an x and y as both $\mathbf{xy}$ and $\mathbf{yx}$. It would be easy to think that they are not like terms. When you put them in the correct alphabetical order then it is clear they are like terms.

List all Polynomials In Descending Order

List the term with the highest power first and then list the other terms in **descending order** of their powers:

$$3x^4 + 5x^3 - 4x^2 + 2x + 9$$

starts with the fourth power listed first and then lists the terms with lower powers in order.

Simplify Polynomials with several different terms

If a polynomial has several terms then all the terms may not be like terms. For example, the expression $3x^2 + 5x + 12x^2 + 8x$ has like terms $\mathbf{5x}$ and $\mathbf{8x}$ and like terms $\mathbf{3x^2}$ and $\mathbf{12x^2}$ but the four terms are not all alike. The $\mathbf{5x}$ and $\mathbf{8x}$ are like terms and can be replaced by $\mathbf{13x}$. The $\mathbf{3x^2}$ and $\mathbf{12x^2}$ are like terms and can be replaced by $\mathbf{15x^2}$ but you cannot combine the $\mathbf{15x^2}$ and the $\mathbf{13x}$. If that is the case then combine the different kinds of like terms separately and list the terms in descending order based on the terms powers.

$$3x^2 + 5x + 12x^2 + 8x$$
$$= 15x^2 + 13x$$

Example 1

$$5x + 4x^2 + 3x + 2x^2$$
for x^2 combime the $4 + 2$
for x combime the $5 + 3$
$$= 6x^2 + 8x$$

Example 2

$$3x + 8 - 7x - 2$$
for x combime the $3 - 7$
combime the constants $8 - 2$
$$= -4x + 6$$

Example 3

$$3y - 5y + 2y^2 - 9y^2$$
for y^2 combime the $2 - 9$
for y combime the $3 - 5$
$$= -7y^2 - 2y$$

Example 4

$$\frac{8}{3}x^2 + 5x - \frac{2}{3}x^2 - 9x$$
for x^2 combime the $\frac{8}{3} - \frac{2}{3}$
for x combime the $5 - 9$
$$= 2x^2 - 4x$$

Example 5

$$-6x^2 - \frac{13}{4}x + 2x^2 + \frac{5}{4}x$$
for x^2 combime the $-6 + 2$
for x combime the $\frac{-13}{4} + \frac{5}{4}$
$$= -4x^2 - 2x$$

Example 6

$$\frac{3}{2}y + \frac{2}{3}y^2 - \frac{7}{2}y - \frac{11}{3}y^2$$
for y^2 combime the $\frac{2}{3} - \frac{11}{3}$
for y combime the $\frac{3}{2} - \frac{7}{2}$
$$= -3y^2 - 2y$$

