Section 4 – 0A: Exponents

The Product Rule

We have seen how an exponent can be used to shorten the way we write repeated products

x^3 means $(x)(x)(x)$ x^2 means $(x)(x)$ x^4 means $(x)(x)(x)(x)$ $3x^2$ means $3(x)(x)$

There are also rules that can simplify our work when we **multiply several terms** with exponents.

Example 1

$\left(x^2\right)\left(x^3\right) =$

$\overbrace{(x \bullet x)(x \bullet x \bullet x)}^{5\ x\text{'s}}$

which equals x^5 so

$\left(x^2\right)\left(x^3\right) = x^{2+3} = x^5$

Example 2

$(x)\left(x^6\right) =$

$\overbrace{(x)(x \bullet x \bullet x \bullet x \bullet x \bullet x)}^{7\ x\text{'s}}$

which equals x^6 so

$(x)\left(x^6\right) = x^{1+6} = x^7$

Example 3

$\left(3x^2\right)\left(5x^4\right) =$

$3 \bullet 5\overbrace{(x \bullet x)(x \bullet x \bullet x \bullet x)}^{6\ x\text{'s}}$

which equals $15x^6$ so

$\left(3x^2\right)\left(5x^4\right) = 3 \cdot 5x^{2+4} = 15x^6$

The examples above suggest that there is a rule for the product of terms with exponents.

$$\left(\mathrm{A}x^{\mathrm{C}}y^{\mathrm{D}}\right)\left(\mathrm{B}x^{\mathrm{E}}y^{\mathrm{F}}\right) = \overbrace{\mathrm{A} \bullet \mathrm{B}} \bullet x^{\overbrace{\mathrm{C+E}}} \bullet y^{\overbrace{\mathrm{D+F}}}$$

The Product Rule

$$\left(\mathrm{A}x^{\mathrm{C}}y^{\mathrm{D}}\right)\left(\mathrm{B}x^{\mathrm{E}}y^{\mathrm{F}}\right) = \mathrm{A} \bullet \mathrm{B} \bullet x^{\mathrm{C+E}} \bullet y^{\mathrm{D+F}}$$

To find the product of 2 variable terms:

1. **Multiply the coefficients** of each term and use that product as the new coefficient.
2. To find the final exponent of each different variable base keep the common base and **add the exponents** of each common base. List the variable bases in alphabetical order.

Example 4

$\left(2y^5\right)\left(3y^6\right)$

$= 2 \cdot 3y^{5+6}$

$= 6y^{11}$

Example 5

$\left(x^2y^3\right)\left(xy^5\right)$

$= x^{2+1}y^{3+5}$

$= x^3y^8$

Example 6

$\left(-2x^4y\right)\left(5xy^5\right)$

$= -2 \cdot 5x^{4+1}y^{1+5}$

$= -10x^5y^6$

The Power Rule

The exponent outside a parentheses tells you how many times to multiply the term inside the parentheses as a product.

Example 1

$$\left(x^2\right)^3 = \left(x^2\right)\left(x^2\right)\left(x^2\right)$$

$$= x^{2+2+2}$$

which equals x^6 so

$$\left(x^2\right)^3 = x^{2\bullet 3} = x^6$$

Example 2

$$\left(xy^3\right)^2 = \left(xy^3\right)\left(xy^3\right)$$

$$= x^{1+1}y^{3+3}$$

which equals x^2y^6 so

$$\left(x^1y^3\right)^2 = x^{1\bullet 2}y^{3\bullet 2} = x^2y^6$$

Example 3

$$\left(2y^4\right)^3 = \left(2y^4\right)\left(2y^4\right)\left(2y^4\right)$$

$$= 2^3y^{4+4+4}$$

which equals $8y^{12}$ so

$$\left(2^1y^4\right)^3 = 2^{1\bullet 3}y^{4\bullet 3} = 8y^{12}$$

The examples above suggest that there is a rule for a term raised to an exponent

$$\left(A^1x^By^C\right)^D = A^{\overbrace{1\bullet D}}x^{\overbrace{B\bullet D}}y^{\overbrace{C\bullet D}}$$

The Power Rule

$$\left(A^1x^By^C\right)^D = A^{1\bullet D}x^{B\bullet D}y^{C\bullet D}$$

To simplify a **variable term** inside a parentheses that has been raised to an **exponent outside the parentheses:**

1. If a constant or variable inside does not have an exponent put a 1 above it.
2. Multiply the exponent **outside** the parentheses by each exponent **inside** the parentheses
3. Simplify any constant that is raised to an exponent. $3^2 = 9$

Example 4

$$\left(x^2y^3\right)^4$$

$$= x^{2\cdot 4}y^{3\cdot 4}$$

$$= x^8y^{16}$$

Example 5

$$\left(5^1x^3y^4\right)^2$$

$$= 5^{1\cdot 2}x^{3\cdot 2}y^{4\cdot 2}$$

$$= 25x^6y^8$$

Example 6

$$\left(2^1xy^5\right)^3$$

$$= 2^3x^{1\cdot 3}y^{5\cdot 3}$$

$$= 8x^3y^{15}$$

The Quotient Rule

The following examples show quotients where **the power of the x term is larger on the top**

Example 1

$$\frac{x^5}{x^2}=\frac{x\bullet x\bullet x\bullet x\bullet x}{x\bullet x}$$

$$=\frac{\cancel{x}\bullet \cancel{x}\bullet x\bullet x\bullet x}{\cancel{x}\bullet \cancel{x}}$$

which equals x^3 so

$$\frac{x^5}{x^2}=x^{5-2}=x^3$$

Example 2

$$\frac{x^6}{x^4}=\frac{x\bullet x\bullet x\bullet x\bullet x\bullet x}{x\bullet x\bullet x\bullet x}$$

$$=\frac{\cancel{x}\bullet \cancel{x}\bullet \cancel{x}\bullet \cancel{x}\bullet x\bullet x}{\cancel{x}\bullet \cancel{x}\bullet \cancel{x}\bullet \cancel{x}}$$

which equals x^2 so

$$\frac{x^6}{x^4}=x^{6-2}=x^2$$

Example 3

$$\frac{6x^4}{4x^3}=\frac{6}{4}\frac{x\bullet x\bullet x\bullet x}{x\bullet x\bullet x}$$

$$=\frac{\cancel{6}^{3}}{\cancel{4}^{2}}\ \frac{\cancel{x}\bullet \cancel{x}\bullet \cancel{x}\bullet x}{\cancel{x}\bullet \cancel{x}\bullet \cancel{x}}$$

which equals $\frac{3x}{2}$ so

$$\frac{6x^4}{4x^3}=\frac{3x^{4-3}}{2}=\frac{3x}{2}$$

The following examples show quotients where **the power of the x term is larger on the bottom**

Example 4

$$\frac{x^4}{x^6}=\frac{x\bullet x\bullet x\bullet x}{x\bullet x\bullet x\bullet x\bullet x\bullet x}$$

$$=\frac{\cancel{x}\bullet \cancel{x}\bullet \cancel{x}\bullet \cancel{x}}{\cancel{x}\bullet \cancel{x}\bullet \cancel{x}\bullet \cancel{x}\bullet x\bullet x}$$

which equals $\frac{1}{x^2}$ so

$$\frac{x^4}{x^6}=\frac{1}{x^{6-4}}=\frac{1}{x^2}$$

Example 5

$$\frac{x^4}{x^5}=\frac{x\bullet x\bullet x\bullet x}{x\bullet x\bullet x\bullet x\bullet x}$$

$$=\frac{\cancel{x}\bullet \cancel{x}\bullet \cancel{x}\bullet \cancel{x}}{\cancel{x}\bullet \cancel{x}\bullet \cancel{x}\bullet \cancel{x}\bullet x}$$

which equals $\frac{1}{x}$ so

$$\frac{x^4}{x^5}=\frac{1}{x^{5-4}}=\frac{1}{x^1}$$

Example 6

$$\frac{8x^3}{6x^5}=\frac{8}{6}\frac{x\bullet x\bullet x}{x\bullet x\bullet x\bullet x\bullet x}$$

$$=\frac{\cancel{8}^{4}}{\cancel{6}^{3}}\frac{\cancel{x}\bullet \cancel{x}\bullet \cancel{x}}{\cancel{x}\bullet \cancel{x}\bullet \cancel{x}\bullet x\bullet x}$$

which equals $\frac{4}{3x^2}$ so

$$\frac{8x^3}{6x^5}=\frac{4}{3x^{5-3}}=\frac{4}{3x^2}$$

The examples above suggest a rule for the quotient of terms with exponents of common bases.

The Quotient Rule for Mononominal terms

If $T>B$

top exponent $>$ bottom exponent

$$\frac{x^T}{x^B}=\frac{x^{T-B}}{1}$$

If $B>T$

bottom exponent $>$ top exponent

$$\frac{x^T}{x^B}=\frac{1}{x^{B-T}}$$

1. Reduce the coefficients by canceling (reducing) the numbers just like a normal fraction
2. **Subtract** the **exponents** of each **common variable** and put the answer where the **largest exponent** for that variable **was**.
3. If a **variable** has equal exponents on the top and bottom of the fraction then that variable and it's exponents are cancelled out

The Power Rule, Product Rule and Quotient Rule Together

The order of operations **PEMDAS** requires that we perform what's inside the parenthesis first, then perform any exponents and then perform multiplication. A fraction bar is considered a parenthesis with the top of the fraction inside one parenthesis and the bottom of the fraction inside another parenthesis. This means that if the Power Rule and the Product Rule and Quotient Rule are in the same problem we perform the **Power Rule first** and then perform the **Product Rule second** on the numerator and then again on the denominator. Finish by performing the **Quotient Rule last.** Not every problem will have all three rules in it. If only two of the three rules are present then perform the ones present in the order listed below.

1. Power Rule First **2. Product Rule Second** **3. Quotient Rule Last**

Example 1

Power Rule

$$\left(3xy^2\right)^2\left(x^2y\right)^3$$

Product Rule

$$=\left(9x^2y^4\right)\left(x^6y^3\right)$$

$$=9x^8y^7$$

Example 2

Power Rule

$$\left(3x^2y^3\right)^2\left(2x^2y\right)$$

Product Rule

$$=\left(9x^4y^6\right)\left(2x^2y\right)$$

$$=18x^6y^7$$

Example 3

Power Rule

$$\left(2xy^2\right)^3\left(3x^2y\right)^2$$

Product Rule

$$=\left(8x^3y^6\right)\left(9x4y^2\right)$$

$$=72x^7y^8$$

Example 4

Product Rule

$$\frac{2x^{10}}{\left(3x^5\right)\left(4x^3\right)}$$

Quotent Rule

$$=\frac{2x^{10}}{12x^8}$$

$$=\frac{x^2}{6}$$

Example 5

Power Rule

$$\frac{\left(5x^4\right)^2}{\left(2x^2\right)^3}$$

Quotent Rule

$$=\frac{25x^8}{8x^6}$$

$$=\frac{25x^2}{8}$$

Example 6

Power Rule

$$\frac{\left(4x^3\right)^2}{\left(2x^5\right)^3}$$

Quotent Rule

$$=\frac{16x^6}{8x^{15}}$$

$$=\frac{2}{x^9}$$

