

Section 3 – 3: Word Problems with Systems of Equations

Many word problems are solved by stating one of the unknowns as x and then stating all the other unknowns in terms of x . The word problems in this section can be best solved by stating one of the unknowns as x and the other as y . After finding 2 equations with x and y in them the system is then solved by either elimination or substitution.

Example 1. The sum of two positive numbers is 14. The difference of the two numbers is 4. Find both of the numbers.

Let one number = x
 the other number = y

Sentence	Meaning	Equation
The sum of two positive numbers is 14	If we add the two numbers the sum is 14	$x + y = 14$
The difference of the two numbers is 4	If we subtract the two numbers the difference is 2	$x - y = 2$

$$\begin{cases} x + y = 14 \\ x - y = 2 \end{cases} \quad \text{add both equations}$$

$$2x = 16 \quad \text{solve for } x$$

$$x = 8$$

plug $x = 8$ into either of the original equations and solve for y

$$8 + y = 14$$

$$y = 6$$

Answer: $x = 8$ and $y = 6$

Check

$$8 + 6 = 14$$

$$8 - 6 = 2$$

Example 2. The sum of two numbers positive is 24. One number is 12 less than three times the other number. Find both of the numbers.

Let one number = x
 the other number = y

Sentence	Meaning	Equation
The sum of two positive numbers is 24	If we add the two numbers ($x + y$) the sum is 24 = 24	$x + y = 24$
One number is 12 less than three times the other number	One number is $x =$ 12 less than three times the other number (y) $3y - 12$	$x = 3y - 12$

$$\begin{cases} x + y = 24 \\ x = 3y - 12 \end{cases} \text{ substitutue the } 3y - 12 \text{ for } x \text{ in the top equation}$$

$$(x) + y = 24 \quad \text{substitutue the } 3y - 12 \text{ for } x$$

$$(3y - 12) + y = 24 \quad \text{solve for } y$$

$$3y - 12 + y = 24$$

$$4y - 12 = 24$$

$$4y = 36$$

$$y = 9$$

plug $y = 9$ into the second of the original equations and solve for x

$$x = 3y - 12 \text{ and } y = 9$$

$$x = 3(9) - 12$$

$$x = 27 - 12$$

$$x = 15$$

Answer: $x = 15$ and $y = 9$

Check

$$15 + 9 = 24$$

$$15 = 3(9) - 12$$

Example 3. The difference of two numbers positive is 12. The largest number is 4 less than three times the smaller number. Find both of the numbers.

Let the largest number = x
the smaller number = y

Note: If both numbers are positive then the difference must be written as largest minus smallest to get a positive 12 as an answer

Sentence	Meaning	Equation
The difference of two positive numbers is 12	If we subtract the two numbers $x - y$ the difference is 12	$x - y = 12$
The largest number is 2 less than three times the other number	the largest number (x) is = 4 less than three times the other number (y) $3y - 4$	$x = 3y - 4$

$$\begin{cases} x - y = 12 \\ x = 3y - 4 \end{cases} \text{ substitutue the } 3y - 4 \text{ for } x \text{ in the top equation}$$

$$(x) + y = 24 \quad \text{substitutue the } 3y - 12 \text{ for } x$$

$$(3y - 4) - y = 12 \quad \text{solve for } y$$

$$3y - 4 - y = 12$$

$$2y - 4 = 12$$

$$2y = 16$$

$$y = 8$$

plug $y = 8$ into the second of the original equations and solve for x

$$x = 3y - 4 \text{ and } y = 8$$

$$x = 3(8) - 4$$

$$x = 20$$

Answer: $x = 20$ and $y = 8$

Check:

$$20 - 8 = 12$$

$$20 = 3(8) - 4$$

Example 4. The total number of doors and windows in a house is 45. The number of windows is 10 more than six times the number of doors. Find the total number of windows and doors.

Let doors = D
 windows = W

Sentence	Meaning	Equation
The total number of doors and windows is 45	If we add the number of doors and windows the sum is 45	$D + W = 45$
The number of windows is 10 more than six times the number of doors	The number of windows (w) is = 10 more than six times the number of doors (d) $6D + 10$	$W = 6D + 10$

$$\begin{cases} D + W = 45 \\ W = 6D + 10 \end{cases} \text{ substitutue the } 6D + 10 \text{ for } W \text{ in the top equation}$$

$$D + (W) = 45 \quad \text{substitutue the } 6D + 10 \text{ for } W$$

$$D + (6D + 10) = 45 \quad \text{solve for } W$$

$$D + 6D + 10 = 45$$

$$7D + 10 = 45$$

$$7D = 35$$

$$D = 5$$

plug $D = 5$ into the second equation and solve for W

$$W = 6D + 10 \text{ and } D = 5$$

$$W = 6(5) + 10$$

$$W = 30 + 10$$

$$W = 40$$

Answer: $D = 5$ and $W = 40$

Check

$$5 + 40 = 45$$

$$40 = 6(5) + 10$$

Example 5. The total number of doors and windows in a house is 11. Three times the number of doors plus 2 times the numbers of windows is 30. Find the total number of windows and doors.

Let doors = D
windows = W

Sentence	Meaning	Equation
The total number of doors and windows is 11	If we add the number of doors and windows the sum is 11	$D + W = 11$
The number of windows is 10 more than six times the number of doors	three times the number of doors (3D) plus 2 times the number of windows (2W) is 30	$3D + 2W = 30$

$$\begin{cases} D + W = 11 \\ \underline{3D + 2W = 30} \end{cases}$$

$$\begin{cases} -3D - 3W = -33 \\ \underline{3D + 2W = 30} \end{cases}$$

$$\begin{cases} -3D - 3W = -33 \\ \underline{3D + 2W = 30} \end{cases}$$

$$-W = -3$$

$$W = 3$$

plug $W = 3$ into $D + W = 11$ and solve for D

$$D + 3 = 11$$

$$D = 8$$

Answer: $D = 8$ and $W = 3$

Check

$$8 + 3 = 11$$

$$3(8) + 2(3) = 30$$