Section 3 – 1C: Solving a System of Equations by Elimination

Solve each system by the elimination method.

Example 1

$$\begin{matrix}\text{Equation A} \\ \text{Equation B}\end{matrix}\begin{cases} 3x+5y=2 \\ -3x+y=-14 \end{cases}$$

Add Equation A and Equation B
to eliminate the x terms

$$\begin{array}{r} 3x+5y=2 \\ \underline{-3x+y=-14} \\ 6y=-12 \\ y=-2 \end{array} \qquad \text{Solve for y}$$

Plug $y=-2$ into either equation A or B
and solve for x

Equation A

$3x+5(-2)=2$

$3x-10=2$

$3x=12$

$x=4$

Answer: (4,–2)

check:

$$\begin{cases} 3(4)+5(-2)=2 \\ -3(4)+(-2)=-14 \end{cases}$$

Example 2

$$\begin{matrix}\text{Equation A} \\ \text{Equation B}\end{matrix}\begin{cases} 3x-2y=3 \\ 8x+2y=30 \end{cases}$$

Add Equation A and Equation B
to eliminate the y terms

$$\begin{array}{l} 3x-2y=2 \\ \underline{5x+2y=30} \\ 8x \quad\; =32 \\ \quad\quad x=4 \end{array} \qquad \text{Solve for x}$$

Plug $x=4$ into either equation A or B
and solve for y

Equation B

$5(4)+2y=30$

$20+2y=30$

$2y=10$

$y=5$

Answer: (4,5)

check:

$$\begin{cases} 3(4)-2(5)=2 \\ 5(4)+2(5)=30 \end{cases}$$

Special Cases: No Solution or All points On The Line

Example 3

Equation A, Equation B:

$$\begin{cases} 2x + 7y = 3 \\ -2x - 7y = 5 \end{cases}$$

Add Equation A and Equation B
to eliminate the x terms

$$\begin{array}{r} 2x + 7y = 3 \\ \underline{-2x - 7y = 5} \\ 0 = 8 \end{array}$$

Stop: Both the x and t terms canceled out
and the remaining eqution $0 = 8$ is false

The lines are parallel,
they have no common points

Answer: No Solution

Example 4

Equation A, Equation B:

$$\begin{cases} -4x - 5y = 3 \\ 4x + 5y = -3 \end{cases}$$

Add Equation A and Equation B
to eliminate the x terms

$$\begin{array}{r} -4x - 5y = 3 \\ \underline{4x + 5y = -3} \\ 0 = 0 \end{array}$$

Stop: Both the x and y terms canceled out
and the remaining eqution $0 = 0$ is true

Both equations discribe the same line
any point on $-4x - 5y = 3$
would also be on $4x + 5y = -3$

Answer: All Points on $4x + 5y = -3$
or
Answer: All Points on $4x - 5y = 3$
either one of the above is correct

Example 5

Equation A, Equation B
$$\begin{cases} 4x + 3y = 16 \\ -2x + y = 2 \end{cases}$$

Multiply Equation B by 2
to eliminate the x terms

$$2\begin{cases} 4x + 3y = 16 \\ -2x + y = 2 \end{cases}$$

$$\begin{array}{r} 4x + 3y = 16 \\ \underline{-4x + 2y = 4} \\ 5y = 20 \\ y = 4 \end{array}$$

Now add the two equations

Solve for y

Plug $y = 4$ into either equation A or B
and solve for x

Equation B

$4x + 3(4) = 16$

$4x + 12 = 16$

$4x = 4$

$x = 1$

Answer: $(1,4)$

check:

$$\begin{cases} 4(1) + 3(4) = 16 \\ -2(1) + (4) = 2 \end{cases} \qquad \begin{cases} 4x + 3y = 16 \\ -2x + y = 2 \end{cases}$$

Example 6

Equation A, Equation B
$$\begin{cases} 3x + y = 2 \\ 2x + 3y = 20 \end{cases}$$

Multiply Equation A by -3
to eliminate the y terms

$$-3\begin{cases} 3x + y = 2 \\ 2x + y = 2 \end{cases}$$

$$\begin{array}{r} -9x - 3y = -6 \\ \underline{2x + 3y = 20} \\ -7x = 14 \\ x = -2 \end{array}$$

Now add the two equations

Solve for x

Plug $x = -2$ into either equation A or B
and solve for y

Equation A

$3(-2) + y) = 2$

$-6 + y = 2$

$y = 8$

Answer: $(-2,8)$

check:

$$\begin{cases} 3(-2) + (8) = 2 \\ 2(-2) + 3(8) = 20 \end{cases} \qquad \begin{cases} 3x + y = 2 \\ 2x + 3y = 20 \end{cases}$$

Example 7

Equation A, Equation B
$$\begin{cases} 2x+2y=7 \\ 4x-3y=-7 \end{cases}$$

Multiply Equation A by -2
to eliminate the x terms

$$\begin{aligned} -2 &\begin{cases} 2x+2y=7 \\ 4x-3y=-7 \end{cases} \end{aligned}$$

$$\begin{aligned} -4x-4y&=-14 \\ 4x-3y&=-7 \\ \hline -7y&=-21 \\ y&=3 \end{aligned}$$

Add the two equations

Solve for y

Plug $y=3$ into either equation A or B
and solve for x

Equation B

$4x-3(3)=-7$

$4x-9=-7$

$4x=2$

$x=\dfrac{1}{2}$

Answer: $\left(\dfrac{1}{2},3\right)$

check:

$$\begin{cases} 2\left(\dfrac{1}{2}\right)+2(3)=7 \\ 4\left(\dfrac{1}{2}\right)-3(3)=-7 \end{cases} \qquad \begin{cases} 2x+2y=7 \\ 4x-3y=-7 \end{cases}$$

Example 8

Equation A, Equation B
$$\begin{cases} 4x-6y=-2 \\ 2x+3y=3 \end{cases}$$

Multiply Equation B by 2
to eliminate the y terms

$$2\begin{cases} 4x-6y=-2 \\ 2x+3y=3 \end{cases}$$

$$\begin{aligned} 4x-6y&=-2 \\ 4x+6y&=6 \\ \hline 8x\quad&=4 \\ x&=\frac{4}{8}=\frac{1}{2} \end{aligned}$$

Add the two equations

Solve for x

Plug $x=\dfrac{1}{2}$ into either equation A or B
and solve for y

Equation A

$4\left(\dfrac{1}{2}\right)-6y=-2$

$2-6y=-2$

$-6y=-4$

$y=\dfrac{-4}{-6}=\dfrac{2}{3}$

Answer: $\left(\dfrac{1}{2},\dfrac{2}{3}\right)$

check:

$$\begin{cases} 4\left(\dfrac{1}{2}\right)-6\left(\dfrac{2}{3}\right))=-2 \\ 2\left(\dfrac{1}{2}\right)+3\left(\dfrac{2}{3}\right)=3 \end{cases} \qquad \begin{cases} 4x-6y=-2 \\ 2x+3y=3 \end{cases}$$

You may need to multiply both equations by different numbers to eliminate one of the variables

Example 9

We will eliminate the x variables

Equation A $\begin{cases} 2x - 3y = 3 \end{cases}$
Equation B $\begin{cases} 3x + 4y = 13 \end{cases}$

$$\begin{cases} 2x - 3y = 3 \\ 3x + 4y = 13 \end{cases}$$

You must mutiply both rows by different numbers to eliminate a variable

Multiply Equation A by -3
Multiply Equation B by 2
to eliminate the x terms

$$\begin{matrix} -3 \\ 2 \end{matrix}\begin{cases} 2x - 3y = 3 \\ 3x + 4y = 13 \end{cases}$$

$-6x + 9y = -9$
$6x + 8y = 26$ Now add the two equations
$17y = 17$ Solve for y
$y = 1$

Plug $y = 1$ into either equation A or B and solve for x

Equation B
$3x + 4(1) = 13$
$3x + 4 = 13$
$3x = 9$
$x = 3$
Answer: $(3,1)$

check:

$$\begin{cases} 2(3) - 3(1) = 3 \\ 3(3) + 4(1) = 13 \end{cases} \qquad \begin{cases} 2x - 3y = 3 \\ 3x + 4y = 13 \end{cases}$$

Example 10

This is the **same problem as Example 9** but we will eliminate the y variables

Equation A, Equation B
$$\begin{cases} 2x - 3y = 3 \\ 3x + 4y = 13 \end{cases}$$

You must mutiply both rows by different numbers to eliminate a variable

Multiply Equation A by 4
Multiply Equation B by 3
to eliminate the y terms

$$\begin{matrix} 4 \\ 3 \end{matrix}\begin{cases} 2x - 3y = 3 \\ 3x + 4y = 13 \end{cases}$$

$8x - 12y = 12$
$9x + 12y = 39$ Now add the two equations
$17x = 51$ Solve for x
$x = 3$

Plug $x = 3$ into either equation A or B and solve for y

Equation A
$2(3) - 3y = 3$
$6 - 3y = 3$
$-3y = -3$
$y = 1$
Answer: $(3,1)$

check:

$$\begin{cases} 2(3) - 3(1) = 3 \\ 3(3) + 4(1) = 13 \end{cases} \qquad \begin{cases} 2x - 3y = 3 \\ 3x + 4y = 13 \end{cases}$$

Example 11

We will eliminate the x variables

$$\begin{cases} \text{Equation A} & -3x+5y=9 \\ \text{Equation B} & 4x+2y=14 \end{cases}$$

You must mutiply both rows by
different numbers to eliminate a variable

Multiply Equation A by 4
Multiply Equation B by 3
to eliminate the x terms

$$\begin{matrix} 4 \\ 3 \end{matrix}\begin{cases} -3x+5y=9 \\ 4x+2y=14 \end{cases}$$

$$\begin{array}{r} -12x+20y=36 \\ \underline{12x+6y=42} \\ 26y=78 \\ y=3 \end{array}$$

Now add the two equations
Solve for y

Plug $y=3$ into either equation A or B
and solve for x

Equation A
$-3x+5(3)=9$
$-3x+15=9$
$-3x=-6$
$x=x$
Answer: $(2,3)$

check:

$$\begin{cases} -3(2)+5(3)=9 \\ 4(2)+2(3)=14 \end{cases} \qquad \begin{cases} -3x+5y=9 \\ 4x+2y=14 \end{cases}$$

Example 12

This is the **same problem as Example 11**
but we will eliminate the y variables

$$\begin{cases} \text{Equation A} & -3x+5y=9 \\ \text{Equation B} & 4x+2y=14 \end{cases}$$

You must mutiply both rows by
different numbers to eliminate a variable

Multiply Equation A by -2
Multiply Equation B by 5
to eliminate the y terms

$$\begin{matrix} -2 \\ 5 \end{matrix}\begin{cases} -3x+5y=9 \\ 4x+2y=14 \end{cases}$$

$$\begin{array}{r} 6x-10y=-18 \\ \underline{20x+10y=70} \\ 26x=52 \\ x=2 \end{array}$$

Now add the two equations
Solve for x

Plug $y=2$ into either equation A or B
and solve for x

Equation A
$-3(2)+5y=9$
$-6+5y=9$
$5y=15$
$y=3$
Answer: $(2,3)$

check:

$$\begin{cases} -3(2)+5(3)=9 \\ 4(2)+2(3)=14 \end{cases} \qquad \begin{cases} -3x+5y=9 \\ 4x+2y=14 \end{cases}$$

Note: Example 11 and 12 solved the exact same system two different way. Example 11 eliminated the x variables first and example 12 solved the same system again by eliminating the y variables. You will never be asked to solve a system both ways as we did above. This was done so you could see it does not matter which variable you chose to eliminate.

What if the system has fractions in the equations?

Eliminate the fractions and get a system without fractions.

Systems with fractions can look overwhelming at first. The key to making these systems easier is to multiply each equation by the LCD (least common denominator) for the fractions in that equation. This will eliminate the denominators and give you a new system with out fraction. Solve this new system like the examples above.

Note: The examples below will not be completely solved. The steps to eliminate the fractions and get a new system without fractions will be shown. The remaining steps to solve the system are left to the student.

Example 13

$$\begin{cases} \text{Equation A} & \dfrac{x}{3} - \dfrac{y}{6} = \dfrac{-2}{3} \\ \text{Equation B} & \dfrac{-x}{2} - \dfrac{y}{4} = 1 \end{cases}$$

First multiply each equation by the LCD for that equation The resulting equations will not contain any fractions.

Multiply Equation A by 6
Multiply Equation B by 3

$$\begin{cases} \text{Equation A} & \dfrac{6}{1}\left(\dfrac{x}{3}\right) - \dfrac{6}{1}\left(\dfrac{y}{6}\right) = \dfrac{6}{1}\left(\dfrac{-2}{3}\right) \\ \text{Equation B} & \dfrac{4}{1}\left(\dfrac{-x}{2}\right) - \dfrac{4}{1}\left(\dfrac{y}{4}\right) = \dfrac{4}{1}\bullet(1) \end{cases}$$

$$\begin{cases} \text{Equation A} & 2x - y = -4 \\ \text{Equation B} & -2x - y = 4 \end{cases}$$

Now you can solve this system like the previous examples.

Example 14

$$\begin{cases} \text{Equation A} & \dfrac{3x}{4} - \dfrac{y}{6} = \dfrac{-3}{8} \\ \text{Equation B} & \dfrac{x}{6} + \dfrac{3y}{4} = 2 \end{cases}$$

First multiply each equation by the LCD for that equation The resulting equations will not contain any fractions.

Multiply Equation A by 24
Multiply Equation B by 12

$$\begin{cases} \text{Equation A} & \dfrac{24}{1}\left(\dfrac{3x}{4}\right) - \dfrac{24}{1}\left(\dfrac{y}{6}\right) = \dfrac{24}{1}\left(\dfrac{-3}{8}\right) \\ \text{Equation B} & \dfrac{24}{1}\left(\dfrac{x}{6}\right) + \dfrac{24}{1}\left(\dfrac{3y}{4}\right) = \dfrac{24}{1}\bullet(2) \end{cases}$$

$$\begin{cases} \text{Equation A} & 18x - 4y = -9 \\ \text{Equation B} & 4x + 18y = 48 \end{cases}$$

Now you can solve this system like the previous examples.

