3 – 1B: Solving a System of Equations by Substitution

1. Look for an equation that has been solved for a single variable. Lets say Equation B
2. Substitute the value of that variable from Equation B into Equation A in place of that variable.
3. Solve the new equation.
4. Plug your answer back into Equation B to find the other variable. Answer with an ordered pair.

Example 1

Equation A $3x + 4y = 44$
Equation B $y = 2x$

Equation B has been solved for y.
Substitute the expression y is equal to (2x) into Equation A in place of y.

Equation A with 2x subtituted for y
$3x + 4(2x) = 44$

Solve for x
$3x + 4(2x) = 44$
$3x + 8x = 44$
$11x = 44$
$x = 4$

plug $x = 4$ into Equation B to find y
$y = 2(4) = 8$

Answer: $(4, 8)$

check:
$3(4) + 4(8) = 44$ $\quad$ $3x + 4y = 44$
$8 = 2(4)$ $\quad$ $y = 2x$

Example 2

Equation A $-2x - y = 27$
Equation B $y = 3x - 2$

Equation B has been solved for y.
Substitute the expression y is equal to $(3x - 2)$ into Equation A in place of y.

Equation A with $3x - 2$ subtituted for y
$-2x - (3x - 2) = 27$

Solve for x
$-2x - (3x - 2) = 27$
$-2x - 3x + 2 = 27$
$-5x + 2 = 27$
$-5x = 25$
$x = -5$

plug $x = -5$ into Equation B to find y
$y = 3(-5) - 2 = -17$

Answer: $(-5, -17)$

check:
$-2(-5) - (-17) = 27$ $\quad$ $-2x - y = 27$
$-17 = 3(-5) - 2$ $\quad$ $y = 3x - 2$

Example 3

Equation A $3x - y = 23$
Equation B $x = -3y + 1$

Equation B has been solved for x.
Substitute the expression x is equal to $(-3y + 1)$
into Equation A in place of the x.

Equation A with $-3y + 1$ subtituted for x
$3(-3y + 1) - y = 23$

Solve for y
$3(-3y + 1) - y = 23$
$-9y + 3 - y = 23$
$-10y + 3 = 23$
$-10y = 20$
$y = -2$

plug $y = -2$ into Equation B to find x
$x = -3(-2) + 1 = 7$

Answer: $(7,-2)$

check:
$3(7) - (-2) = 23$ $\quad$ $3x - y = 23$
$7 = -3(-2) + 1$ $\quad$ $x = -3y + 1$

Example 4

Equation A $-2x + 6y = 8$
Equation B $x = 4y - 3$

Equation B has been solved for x.
Substitute the expression x is equal to $4y - 3$
into Equation A in place of the x.

Equation A with $4y - 3$ subtituted for x
$-2(4y - 3) + 6y = 8$

Solve for y
$-2(4y - 3) + 6y = 8$
$-8y + 6 + 6y = 8$
$-2y + 6 = 8$
$-2y = 2$
$y = -1$

plug $y = -1$ into Equation B to find x
$x = 4(-1) - 3 = -7$

Answer: $(-7,-1)$

check:
$-2(-7) + 6(-1) = 8$ $\quad$ $-2x + 6y = 8$
$-7 = 4(-1) - 3$ $\quad$ $x = 4y - 3$

Example 5

Equation A $2x - y = -6$
Equation B $y = 4x + 3$

Equation B has been solved for y.
Substitute the expression y is equal to $(4x + 3)$ into Equation A in place of y.

Equation A with $4x + 3$ subtituted for y
$2x - (4x + 3) = -6$

Solve for x
$2x - (4x + 3) = -6$
$2x - 4x - 3 = -6$
$-2x - 3 = -6$
$-2x = -3$
$x = \frac{-3}{-2} = \frac{3}{2}$

plug $x = \frac{3}{2}$ into Equation B to find y
$y = \frac{4}{1}\left(\frac{3}{2}\right) + 3 = 9$

Answer: $\left(\frac{3}{2}, 9\right)$

check:
$\begin{cases} 2\left(\frac{3}{2}\right) - (9) = -6 \\ 9 = \frac{4}{1}\left(\frac{3}{2}\right) + 3 \end{cases}$ $\begin{cases} 2x - y = -6 \\ y = 4x + 3 \end{cases}$

Example 6

Equation A $-x + 3y = 3$
Equation B $x = 6y - 4$

Equation B has been solved for x.
Substitute the expression y is equal to $(6y - 4)$ into Equation A in place of x.

Equation A with $6y - 4$ subtituted for x
$-(6y - 4) + 3y = 3$

Solve for y
$-(6y - 4) + 3y = 3$
$-6y + 4 + 3y = 3$
$-3y + 4 = 3$
$-3y = -1$
$x = \frac{-1}{-3} = \frac{1}{3}$

plug $y = \frac{1}{3}$ into Equation B to find x
$x = \frac{6}{1}\left(\frac{1}{3}\right) - 4 = -2$

Answer: $\left(-2, \frac{1}{3}\right)$

check:
$\begin{cases} -(-2) + \frac{3}{1}\left(\frac{1}{3}\right) = 3 \\ -2 = \frac{6}{1}\left(\frac{1}{3}\right) - 4 \end{cases}$ $\begin{cases} -x + 3y = 3 \\ x = 6y - 4 \end{cases}$

Special Cases: No Solution or All points On The Line

Example 7

Equation A $2x - y = -6$
Equation B $y = 2x + 3$

Equation B has been solved for y.
Substitute the expression y is equal to $(2x + 3)$
into Equation A in place of y.

Equation A with $2x + 3$ subtituted for y
$2x - (2x + 3) = -6$

Solve for x
$2x - (2x + 3) = -6$
$2x - 2x - 3 = -6$
$-3 = -6$

Stop: The x term canceled out and
the remaining eqution $-3 = 8$ is false

The lines are parallel,
they have no common points

Answer: No Solution

Example 8

Equation A $2x - 6y = -8$
Equation B $x = 3y - 4$

Equation B has been solved for x.
Substitute the expression x is equal to $(3y - 4)$
into Equation A in place of y.

Equation A with $3y - 4$ subtituted for x
$2(3y - 4) - 6y = 8$

Solve for y
$2(3y - 4) - 6y = -8$
$6y - 8 - 6y = -8$
$-8 = -8$

Stop: The x term canceled out and
the remaining eqution $-8 = -8$ is true

Both equations discribe the same line
any point on $2x - 6y = -8$
would also be on $x = 3y - 4$

Answer: All Points on $2x - 6y = -8$
or
Answer: All Points on $x = 3y - 4$
either one of the above is correct

What if the system given does not have one of the equations solved for one of the variables?

1. Look for a variable in one of the equations that can be solved for without introducing fractions.
2. Solve for that variable.
3. Replace the old equation with the new one.
4. You now have a problem like the others we have already solved. Solve by substitution.

Example 9

Equation A $2x + 3y = 23$
Equation B $2x - y = 4$

First solve Equation B for y

$2x - y = 4$
$-y = 2x + 4$
$y = -2x - 4$

Replace the new equation for Equation B

Equation A $2x + 3y = 23$
Equation B $y = -2x - 4$

We now have the problem set up for substitution. We can now solve it like we did for the previous problems.

Example 10

Equation A $2x + 3y = 23$
Equation B $-x - 3y = -9$

First solve Equation B for x

$-x - 3y = -9$
$-x = 3y - 9$

Replace the new equation for Equation B

Equation A $2x + 3y = 23$
Equation B $x = -3x + 9$

We now have the problem set up for substitution. We can now solve it like we did for the previous problems.

What if no variable in the system can be solved for without getting fractions.

You could go ahead and solve for a variable and get the fraction. You would the have to substitute the expression with the fraction into the other equation getting an even messier equation with fractions. You would then need to solve that equation and plug that answer back in to the other equation to find the other part of the answer.

OR

you could decide that substitution is not a good way to solve systems of this type and use the elimination method from the other section.

It sounds to me like the second idea is the best one unless you love fractions.

