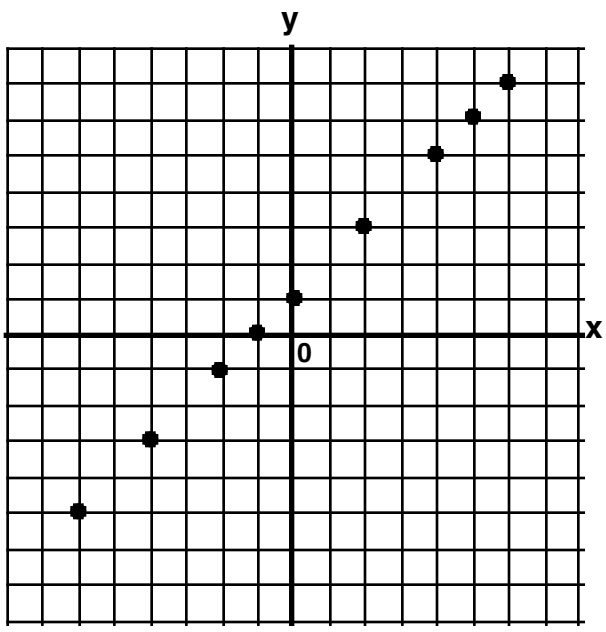
Section 2 – 2: Graphing Linear Equations Introduction

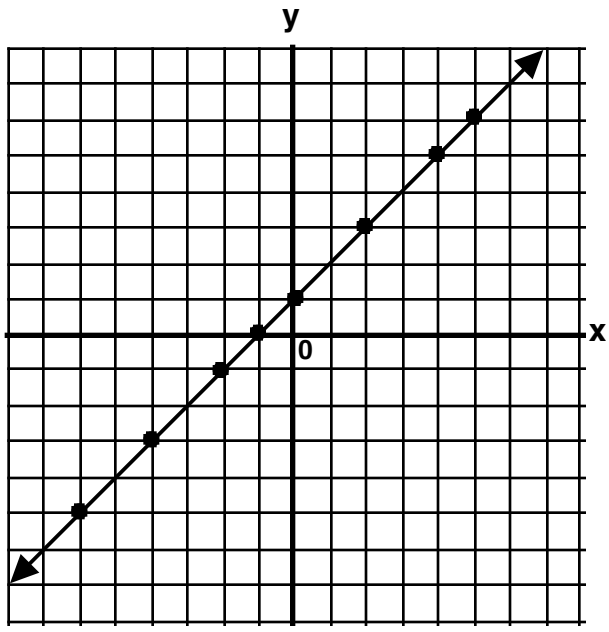
There are an infinite number of ordered pairs that are solutions for equations like **y = x + 1** In fact for every x value you select there is a unique y value that makes that ordered pair (x, y) a solution to the equation. It is not possible to list all the infinite number of ordered pairs that are solutions but we could plot several of them on a coordinate grid and see if a pattern develops.

The points in the table to the right are some of many solutions to **y = x + 1**

x	–6	–4	–2	–1	0	2	4	5
y	–5	–3	–1	0	1	3	5	6

When we graph the points on the coordinate grid below we see that they all seem to lie on a single straight line. The graph to the right of the points shows a line through these points.





When we draw a straight line through the points we see that other points like (1, 2) and (–5, –4) are also on the line and are also solutions to y = x + 1 The line has points on each end to show it continues on in both directions. All the other points the line goes through will have x and y values that are also solutions to the equation y = x + 1.

The line in the graph above is called the graph of **y = x + 1** and the process of finding the ordered pairs and drawing the line is called graphing a linear equation.

To graph a line given a linear equation

1. Select several x values (we will use 3).
2. Find the value of y for each given x.
3. Plot the points on a coordinate grid.
4. Draw a line through the points with an arrow on each end.

Graphing a Linear Equation by finding 3 points on the Line

1. Select 3 x values.

You can chose any three values for x and use them to find the y values for each x value. It is common to chose x =1 , x = 2 and x = 3 to keep the y values as small as possible. Since all ordered pairs (x,y) must be on the graph any 3 values of x will produce 3 points the will determine the same graph for the line. At times a negative x value will be used instead of a positive x value to keep the values for y on the coordinate grid. Most of the grids in this book are –7 to 7 for the x and y values.

Example 1

$y = 4x - 3$ use x = 0, 1, 2

Example 2

$y = -x + 4$ use x = 0, 1, 2

If there is a fraction in the equation then we chose x = 0 for the fist choice of x. The other 2 values of x should be multiples of the fractions denominator. This will ensure the y values are not fractions.

Example 3

$y = \frac{2}{3}x - 4$ use x = 0, 3, 6

Example 4

$y = \frac{-4}{5}x - 1$ use x = – 5, 0, 5

we don't pick x = 10 because the graph does not go beyond 8 for the x values

2. Find the value of y for each given x.

Plug the values for x you have selected into the linear equation and find the corresponding values for y. Do this for all three values of x. This will give you 3 ordered pairs (x,y)

3. Plot the points on a coordinate grid.

If you have correctly found the y values for each x then the 3 points will be on a single straight line. If the points are not on a straight line then recheck the work that was used to find each y value. Also recheck that you graphed the ordered pairs correctly.

4. Draw a line through the points with an arrow on both ends.

Use a straight edge to make the line so it is a clean straight line. The arrows at each end represent the fact that there are an endless number of ordered pairs that make an equation true. All the other points the line goes through will have x and y values that are also solutions to the equation y = x + 1.

Example 1A. Graph **y = 2x – 5 use x = 0 , x = 1 and x = 2**

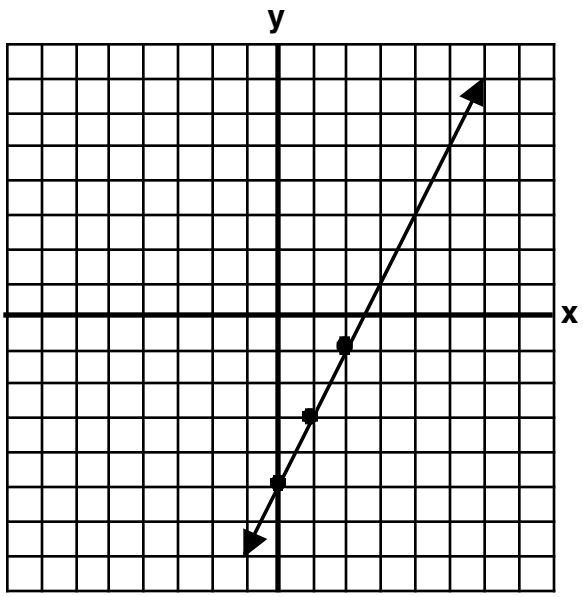
1. For each x value shown (0, 2, and 4) find the y value for each x and complete the table.

X	Y
0	
1	
2	

$2\,(0) - 5 = -5$

$2\,(1) - 5 = -3$

$2\,(2) - 5 = -1$



2. The completed table is shown below.

X	Y
0	–5
1	–3
2	–1

3. Plot the points on a coordinate grid. (0, –5) (1, –3) (2 , –1)

4. Draw a line through the points with an arrow on both ends.

Example 1B. Graph **y = 2x – 5 use x = –1 , x = 3 and x = 5**

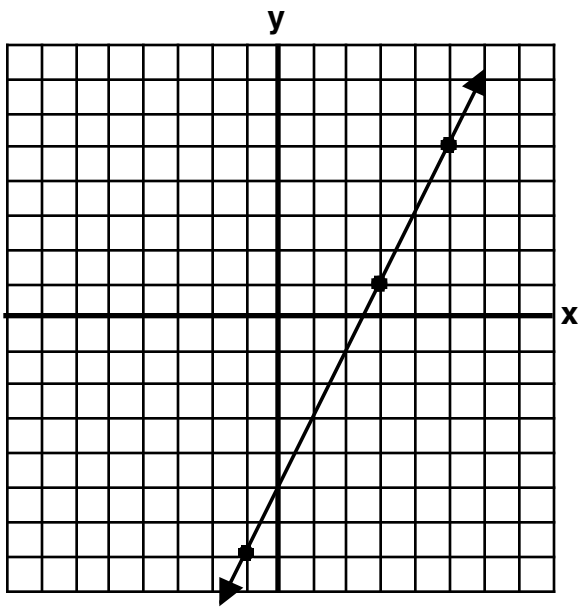
1. For each x value shown (0, 2, and 4) find the y value for each x and complete the table.

X	Y
–1	
3	
5	

$2\,(-1) - 5 = -7$

$2\,(3) - 5 = 1$

$2\,(5) - 5 = 5$



2. The completed table is shown below.

X	Y
–1	–7
3	1
5	5

3. Plot the points on a coordinate grid. (–1, –7) (3, 1) (5 , 5)

4. Draw a line through the points with an arrow on both ends.

You can select any three x values to determine the graph of a line.

The graph of the line represents all of the (x, y) pairs that make the linear equation true. All ordered pairs (x,y) must be on the graph of the line so any 3 values of x will produce 3 points that will determine the same graph for the linear equation.

Example 1A

Graph **y = 2x – 5**
use x = 0 , x = 1 and x = 2

The completed table for

x = – 1 x= 3 and x = 5

X	Y
0	**–5**
1	**–3**
2	**–1**

Example 1B

Graph **y = 2x – 5**
use x = –1 , x = 3 and x = 5

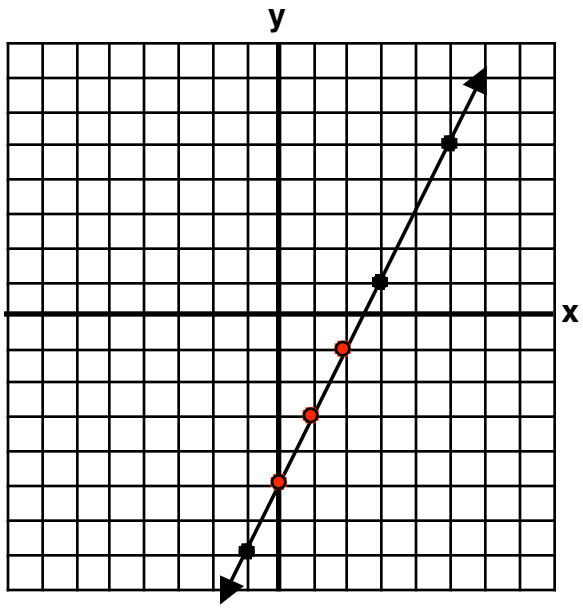
The completed table for

x = – 1 x= 3 and x = 5

X	Y
–1	–7
3	1
5	5

The 3 points for **y = 2x – 5 using x = 0 , x = 1 and x = 2** are graphed in black
and
The 3 points for **y = 2x – 5 using x = – 1 x= 3 and x = 5 are graphed in red.**

Each sets of points determine the same line.



Example 2. Graph **y = –3x + 5 use x = 0 , x = 1 and x = 2**

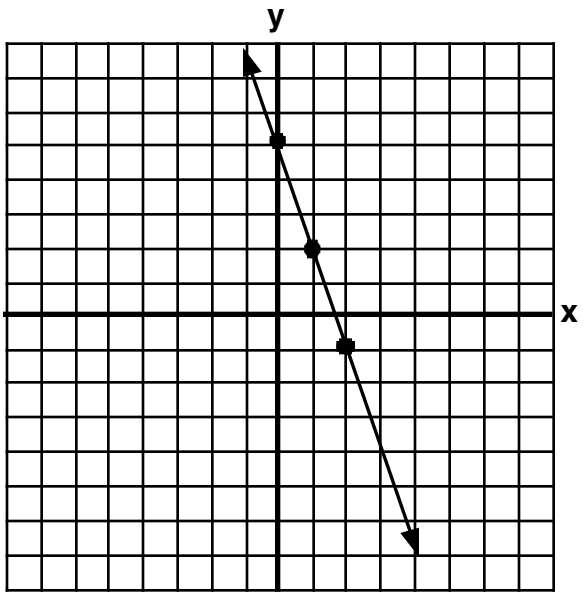
1. For each x value shown (0, 2, and 3) find the y value for each x and complete the table.

X	Y
0	
1	
2	

$-3\,(0) + 5 = 5$

$-3\,(1) + 5 = 2$

$-3\,(2) + 5 = -1$



2. The completed table is shown below.

X	Y
0	5
1	2
2	–1

3. Plot the points on a coordinate grid. (0, 5) (1, 2) (3 , –1)

4. Draw a line through the points with an arrow on both ends.

Example 3. Graph **y = –2x use x = –3 , x = 0 and x = 3**

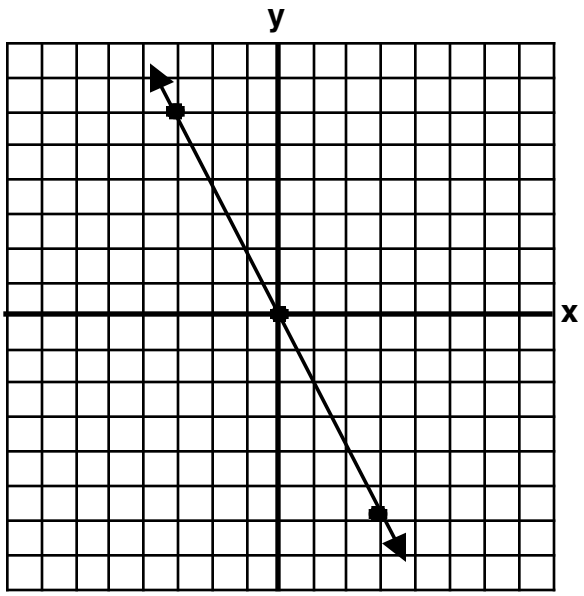
1. For each x value shown (0, 3, and –3) find the y value for each x and complete the table.

X	Y
–3	
0	
3	

$-2\,(-3) = 6$

$-2\,(0) = 0$

$-2\,(3) = -6$



2. The completed table is shown below.

X	Y
–3	6
0	0
3	–6

3. Plot the points on a coordinate grid. (–3, 6) (0, 0) (3 , –6)

4. Draw a line through the points with an arrow on both ends.

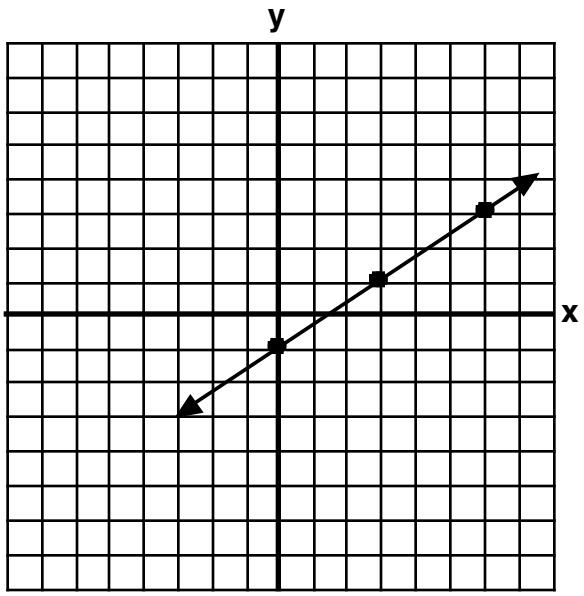
Example 4. Graph $y\frac{2}{3}x-1$ **use x = 0 , x = 3 and x = 6**

X	Y
0	
3	
6	

$\frac{2}{3}(0) - 1 = -1$

$\frac{2}{3}(3) - 1 = 1$

$\frac{2}{3}\ 6 - 1 = 3$



2. The completed table is shown below.

X	Y
0	–1
3	1
6	3

3. Plot the points on a coordinate grid. (0, –1) (3, 1) (6 , 3)

4. Draw a line through the points with an arrow on both ends.

Example 5. Graph $y = \frac{-3}{4}x-2$ **use x = 4 , x = 0 and x = 4**

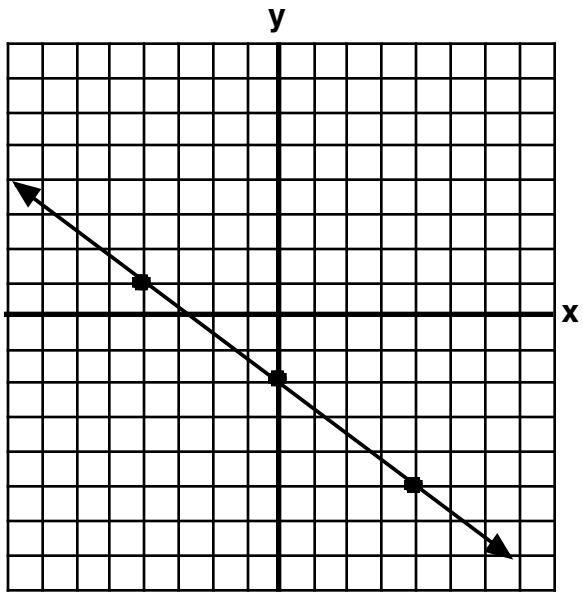
1. For each x value shown (0, 4, and –4) find the y value for each x and complete the table.

X	Y
–4	
0	
4	

$\frac{-3}{4}(-4) - 2 = 1$

$\frac{-3}{4}(0) - 2 = -2$

$\frac{-3}{4}(4) - 2 = -5$



2. The completed table is shown below.

X	Y
–4	1
0	–2
4	–5

3. Plot the points on a coordinate grid. (–4, 1) (0, –2) (4 , –5)

4. Draw a line through the points with an arrow on both ends.

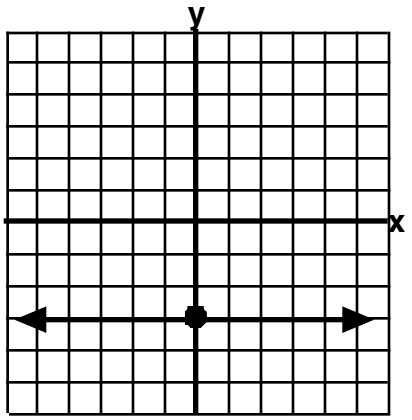
Graphing the Equation of a Horizontal Line

Case 1: If the equation is of the form $y = y_1$ (where y_1 is the constant)

Graph a **horizontal line** crossing the y axis at y_1 (the constant)

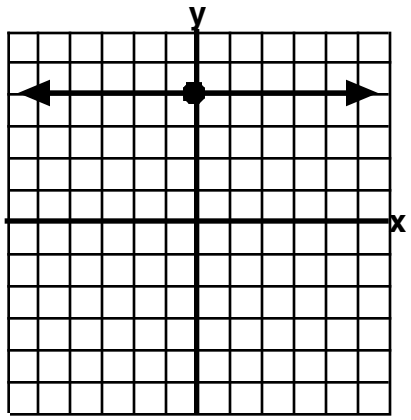
Example 1: Graph $y = -3$

$y = -3$
is graphed as a
horizontal line
through the y axis at –3



Example 2; Graph $y = 4$

$y = 4$
is graphed as a
horizontal line
through the y axis at 4



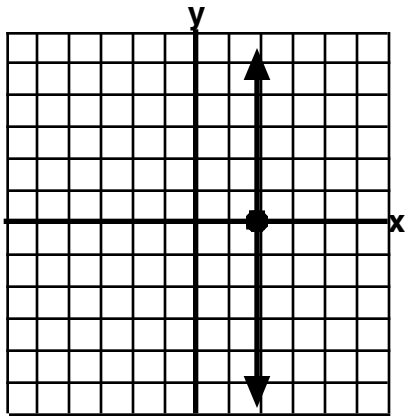
Graphing the Equation of a Vertical Line

Case 2: If the equation is of the form $x = x_1$ (where x_1 is the constant)

Graph a vertical line crossing the x axis at x_1 (the constant)

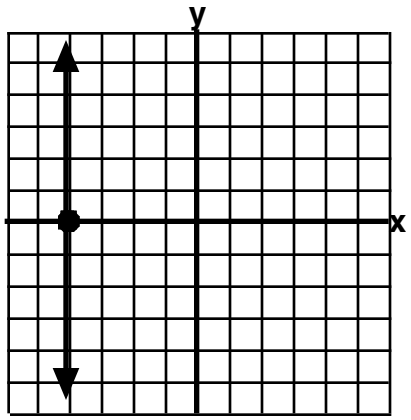
Example 3: Graph $x = 2$

$x = 2$
is graphed as a
vertical line
through the x axis at 2



Example 4; Graph $x = -4$

$x = -4$
is graphed as a
vertical line
through the x axis at –4



Finding the **y-intercept**

The **y-intercept** will be a point on the **y axis**. All points on the y axis have an **x coordinate of 0** (you did not move right or left in the x direction and then moved up or down in the y direction. If we know that the x coordinate will be 0 then to find the y-intercept **replace x with 0** and solve for y.

Finding the y-intercept: To find the y-intercept replace **x with 0** and solve for y.

Example 1

Find the y-intercept for $2x + 3y = 6$

$2(0) + 3y = 6$ (**replace x with a 0**)

$0 + 3y = 6$ ($2(0) = 0$)

$3y = 6$ (solve for y, divide by 3)

$y = 2$

The ordered pair will have an x value of 0

The **y-intercept is (0, 2)**

Example 2

Find the y-intercept for $3x – 4y = 12$

$3(0) – 4y = 12$ (**replace x with a 0**)

$0 – 4y = 12$ ($3(0) = 0$)

$–4y = 12$ (solve for y, divide by –4)

$y = –3$

The ordered pair will have an x value of 0

The **y-intercept is (0, –3)**

Finding the **x-intercept**

The **x-intercept** will be a point on the **x axis**. All points on the x axis have an **y coordinate of 0** (you did not move up or down in the y direction and then moved right or left in the x direction. If we know that the y coordinate will be 0 then to find the x-intercept we **replace y with 0** and solve for x.

Finding the x-intercept: To find the x-intercept **replace y with 0** and solve for . The ordered pair will have a value for y of 0.

Example 1

Find the x-intercept for $2x + 3y = 6$

$2x + 3(0) = 6$ (**replace y with a 0**)

$2x + 0 = 6$ ($3(0) = 0$)

$2x = 6$ (solve for x, divide by 2)

$x = 3$

The ordered pair will have an y value of 0

The **x-intercept is (3, 0)**

Example 2

Find the x-intercept for $3x – 4y = 12$

$3x – 4(0) = 12$ (**replace y with a 0**)

$3x – 0 = 12$ ($3(0) = 0$)

$3x = 12$ (solve for x, divide by 3)

$x = 4$

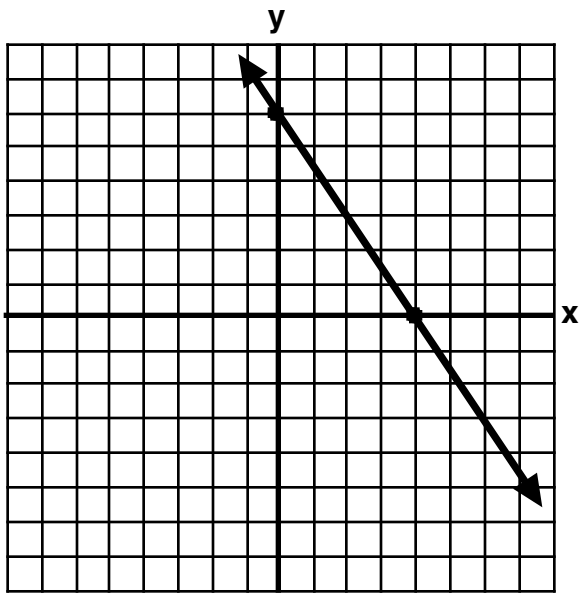
The ordered pair will have an y value of 0

The **x-intercept is (4, 0)**

Graphing Linear Equations for Ax + By = C with x and y Intercepts

Example 1. Graph $3x + 2y = 12$

X	Y	$3x + 2y = 12$
0		Let x = 0 to find the y intercept
	0	Let y = 0 to find the x intercept



If x = 0 then

2y =12

y = 6

If y = 0 then

3x = 12

x = 4

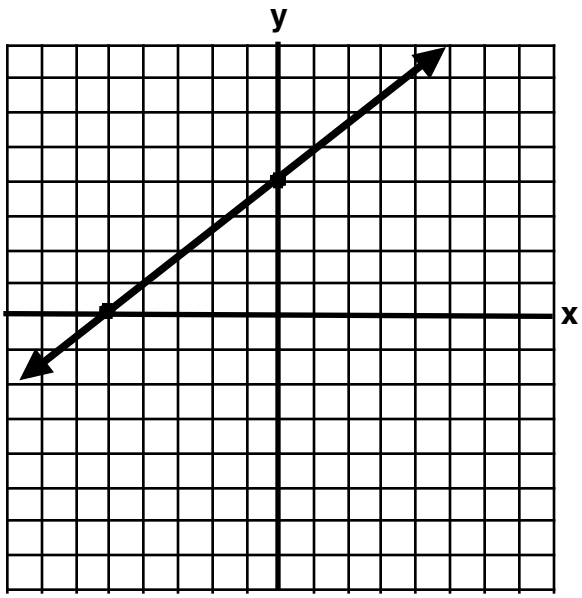
The completed table is shown below

X	Y
0	6
4	0

Plot the points (0 , 6) and (4 , 0) on the coordinate grid above and draw a line through the points with an arrow on both ends.

Example 2. Graph $-4x + 5y = 20$

X	Y	$-4x + 5y = 20$
0		Let x = 0 to find the y intercept
	0	Let y = 0 to find the x intercept



If x = 0 then

5y = 20

y = 4

If y = 0 then

–4x = 20

x = –5

The completed table is shown below

X	Y
0	4
–5	0

Plot the points (0 , 4) and (–5 , 0) on the coordinate grid above and draw a line through the points with an arrow on both ends.

Graphing a Line Given its Equation

There are three different kinds of line graphs possible and each type of graph corresponds to one of three different types of line equations.

Case 1

Equations of the form
y = a constant

like $y = 2$

have
a zero slope

and are graphed as a
horizontal line
through the y axis at 2

Case 2

Equations of the form
x = a constant

like $x = 4$

have
an undefined slope

and are graphed as a
vertical line
through the x axis at 4

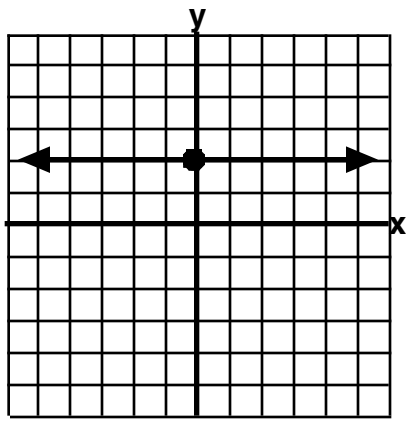
Case 3

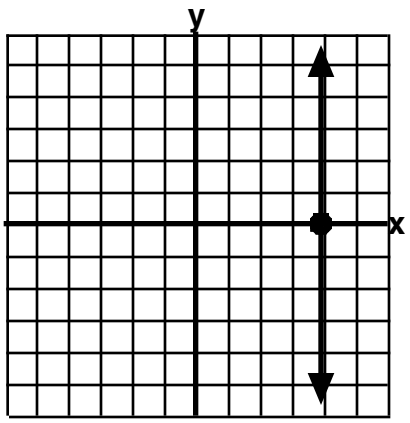
Equations of the form
$y = mx - b$

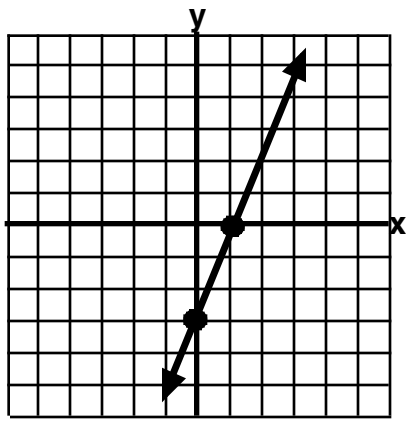
like
$y = 3x - 5$ or
$y = -2x + 4$

have
a slope m that is a non zero number

and are graphed as a
through the y intercept
with a given slope







To Graph a Line given it's Equation

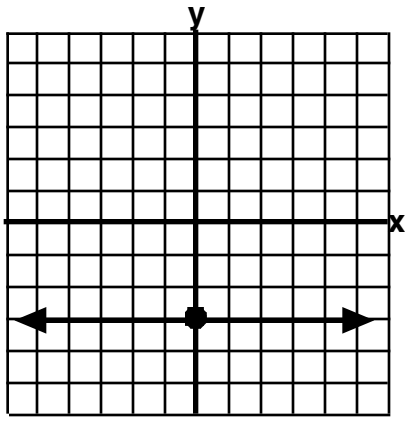
Case 1: If the equation is of the form $y = y_1$ (where y_1 is the constant)

Graph a **horizontal line** crossing the y axis at y_1 (the constant)

Example 1

Graph $y = -3$

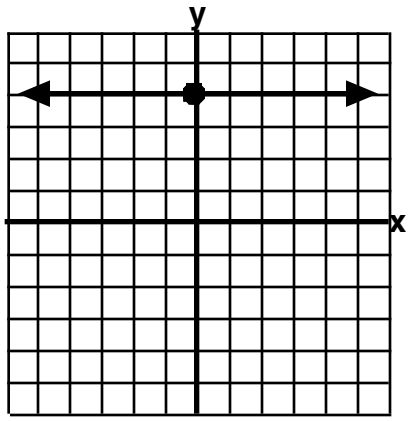
$y = -3$
is graphed as a
horizontal line
through the y axis at –3



Example 2

Graph $y = 4$

$y = 4$
is graphed as a
horizontal line
through the y axis at 4



To Graph a Line given it's Equation

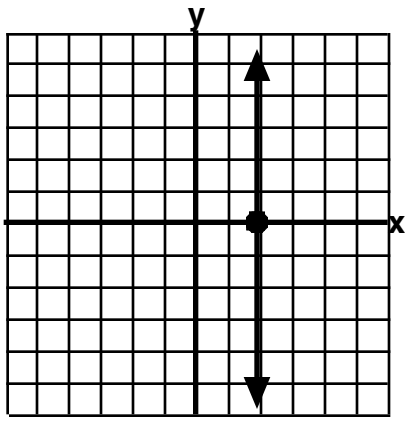
Case 2: If the equation is of the form $x = x_1$ (where x_1 is the constant)

Graph a vertical line crossing the x axis at x_1 (the constant)

Example 1

Graph $x = 2$

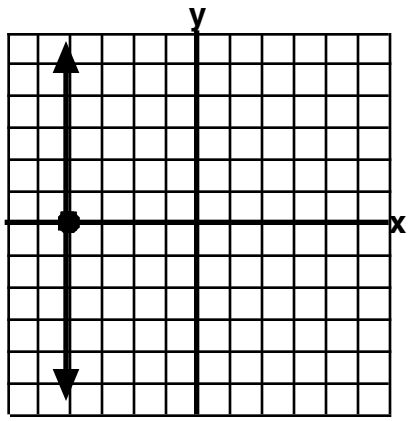
$x = 2$
is graphed as a
vertical line
through the x axis at 2



Example 2

Graph $x = -4$

$x = -4$
is graphed as a
vertical line
through the x axis at –4



To Graph a Line given it's Equation

Case 3: If the equation has x and y variables and can be written in the form $y = mx + b$

Step 1. Put the equation into the form $y = mx + b$

Step 2. List the slope m and the y intercept b

Step 3. plot a point on the graph on the y axis at b.

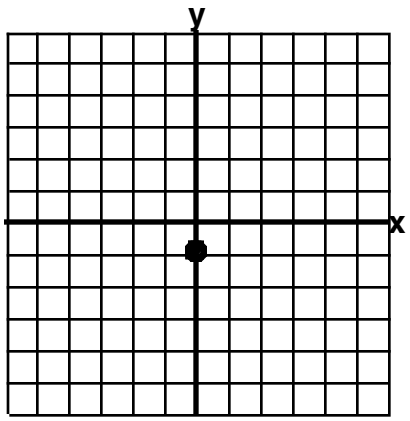
Step 4. To get a second point, start at the y intercept and move in the x and y directions based on the slope.

Step 5. Draw a line through the two points.

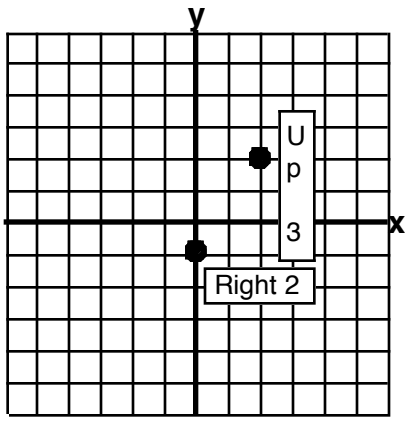
Example 1. Graph $y = \frac{3}{2}x - 1$

Step 1. The slope is $m = \frac{3}{2}$ $\begin{matrix}\text{up } 3 \\ \text{right } 2\end{matrix}$ and the y intercept b is –1

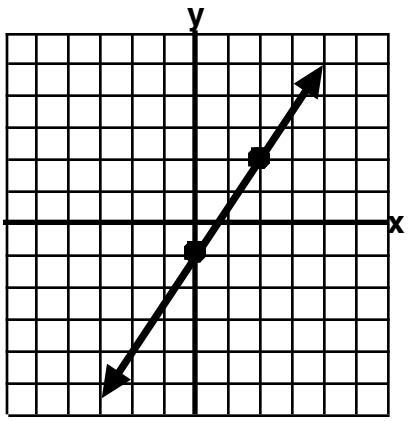
Step 2. Plot (0 , –1)



Step 3. go right 2 up 3 and plot a point



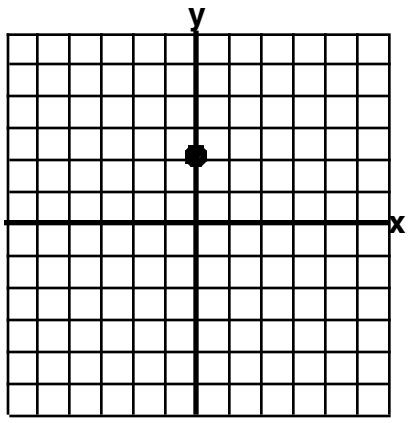
Step 4. Draw the line graph through the 2 points



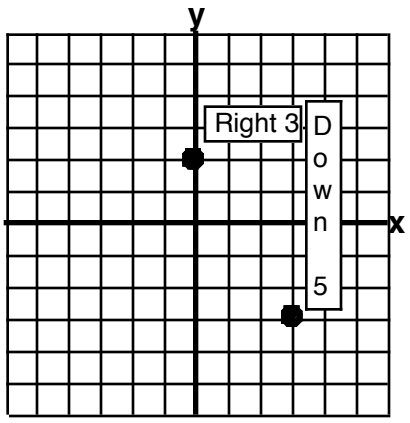
Example 2. Graph $y = \frac{-5}{3}x + 2$

Step 1. The slope is $m = \frac{-5}{3}$ $\begin{matrix}\text{down } 5 \\ \text{right } 3\end{matrix}$ and the y intercept b is 2

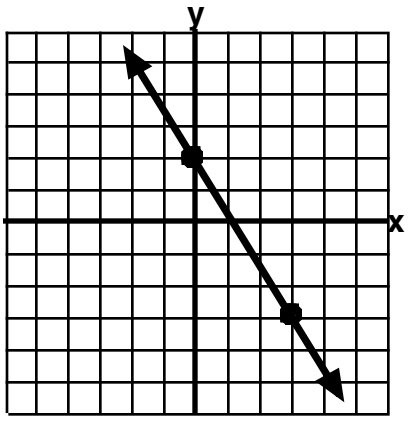
Step 2. Plot (0 , –1)



Step 3. go right 3 down 5 and plot a point



Step 4. Draw the line graph through the 2 points



Example 3. Graph $5x - 2y = 0$

Step 1. Solve for y:

$$5x + 2y = 0$$
$$-5x \qquad -5x$$

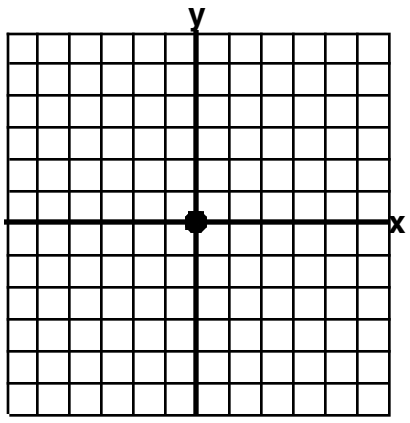
$$2y = -5x$$

$$\frac{2y}{2} = \frac{-5x}{2}$$

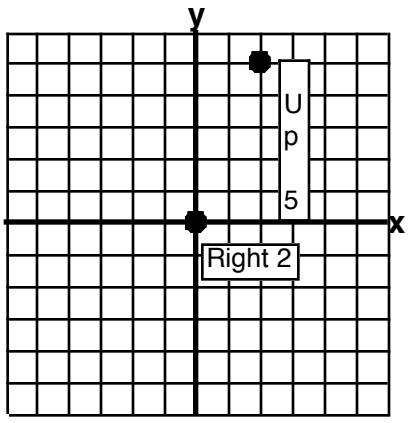
$$y = \frac{5}{2}x$$

Step 2. The slope is $m = \frac{5}{2}$ $\begin{matrix}\text{up } 5 \\ \text{right } 2\end{matrix}$ and the y intercept b is 0

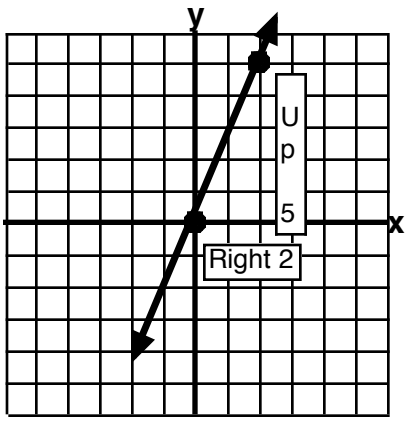
Step 3. Plot (0 , 0)



Step 4. go right 2 up 5 and plot a point



Step 5. Draw the line graph through the 2 points



Example 4. Graph $4x - 3y = -6$

Step 1. Solve for y:

$$4x - 3y = 6$$
$$-4x \qquad -4x$$

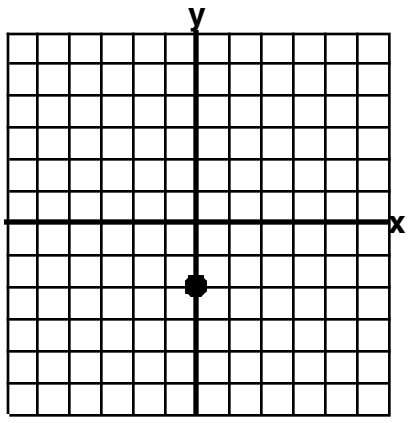
$$-3y = -4x + 6$$

$$\frac{-3y}{-3} = \frac{-4x}{-3} + \frac{6}{-3}$$

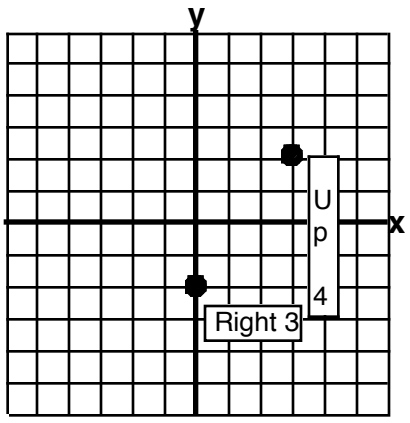
$$y = \frac{4}{3}x - 2$$

Step 2. The slope is $m = \frac{4}{3}$ $\begin{matrix}\text{up } 4 \\ \text{right } 3\end{matrix}$ and the y intercept b is –2

Step 3. Plot (0 , –2)



Step 4. go right 3 up 4 and plot a point



Step 5. Draw the line graph through the 2 points