

## Section 1 – 6:

## Solving Absolute Inequalities

### Less Than Cases

**Case 1: The absolute value is alone and is less than a positive number or less than or equal to a positive number**

If  $|ax + b|$  is alone and  
less than a positive number

$$\begin{aligned} \text{If } |ax + b| < c \\ \text{then} \\ -c < ax + b < c \end{aligned}$$

put whats inside the brackets between  
– c and c

If  $|ax + b|$  is alone and  
less than or equal to a positive number

$$\begin{aligned} \text{If } |ax + b| \leq c \\ \text{then} \\ -c \leq ax + b \leq c \end{aligned}$$

put whats inside the brackets between  
– c and c  
and – c and c are included

#### Example 1

Solve for x  
 $|3x - 12| < 21$

$$-21 < 3x - 12 < 21$$

$$\begin{array}{r} -21 < 3x - 12 < 21 \\ +12 \quad +12 \quad +12 \end{array} \quad \text{add 12 to all 3 parts}$$

$$-9 < 3x < 33 \quad \text{divide all 3 parts by 3}$$

$$-3 < x < 11$$

The graph is the set of points  
between –3 and 11  
but the endpoints are NOT included



$$x \in (-3, 11)$$

#### Example 2

Solve for x  
 $|2x + 8| \leq 4$

$$-4 \leq 2x + 8 \leq 4$$

$$\begin{array}{r} -4 \leq 2x + 8 \leq 4 \\ -8 \quad -8 \quad -8 \end{array} \quad \text{subtract 8 from all 3 parts}$$

$$-12 \leq 2x \leq -4 \quad \text{divide all 3 parts by 2}$$

$$-6 \leq x \leq -2$$

The graph is the set of points  
between –6 and –2  
and the endpoints ARE included



$$x \in [-6, -2]$$

### Example 3

Solve for x

$$|-4x - 8| < 12$$

$$-12 < -4x - 8 < 12 \quad \text{add 8 to all 3 parts}$$

$$\begin{array}{r} -12 < -4x - 8 < 12 \\ +8 \quad \quad +8 \quad +8 \end{array}$$

$$-4 < -4x < 20 \quad \text{divide all 3 parts by } -4$$

you must switch the direction of the inequality

$$1 > x > -5$$

now you must rewrite the statement with  
the smallest number on the left

$$-5 < x < 1$$

The graph is the set of points  
between  $-5$  and  $1$   
but the endpoints are NOT included



$$x \in (-5, 1)$$

### Example 4

Solve for x

$$|-3x + 18| \leq 6$$

$$-6 \leq -3x + 18 \leq 6 \quad \text{subtract 18 from all 3 parts}$$

$$\begin{array}{r} -6 \leq -3x + 18 \leq 6 \\ -18 \quad \quad -18 \quad -18 \end{array}$$

$$-24 \leq -3x \leq -12 \quad \text{divide all 3 parts by } -3$$

you must switch the direction of the inequality

$$8 \geq x \geq 4$$

now you must rewrite the statement with  
the smallest number on the left

$$4 \leq x \leq 8$$

The graph is the set of points  
between  $4$  and  $8$   
and the endpoints ARE included



$$x \in [4, 8]$$

**Case 2: If the absolute value is alone and is less than a negative number or less than or equal to a negative number then the equation has no solution.**

if  
 $|ax + b| < \text{negative number}$

than there is no solution

**Example 5**

Solve for x

$$|2x - 7| < -5$$

$|2x - 7|$  is ALWAYS POSITIVE  
so it is NEVER less than  $-7$

No Solution

if  
 $|ax + b| \leq \text{negative number}$

than there is no solution

**Example 6**

Solve for x

$$|-2x + 3| + 9 \leq 2$$

$$|-2x + 3| \leq -7$$

$|-2x + 3|$  is ALWAYS POSITIVE  
so it is NEVER less than  $-7$

No Solution

**We do not use a graph if there are no numbers that work. We just write No Sol.**

## Greater Than Cases

**Case 3: The absolute value is alone and is greater than a positive number or greater than or equal to a positive number**

If  $|ax + b|$  is alone and greater than a positive number  $c$

If  $|ax + b| > c$   
 then  
 $ax + b > c$  or  $ax + b < -c$   
 greater than  $c$  less than  $-c$

If  $|ax + b|$  is alone and greater than or equal to a positive number  $c$

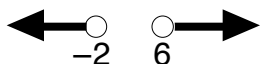
If  $|ax + b| \geq c$   
 then  
 $ax + b \geq c$  or  $ax + b \leq -c$   
 greater than or equal to  $c$  less than or equal to  $-c$

### Example 7

Solve for  $x$   
 $|3x - 6| > 12$

greater than 12 or less than  $-12$   
 $3x - 6 > 12$  or  $3x - 6 < -12$   
 $3x - 6 > 12$        $3x - 6 < -12$   
  
 $3x > 18$        $3x < -6$   
  
 $x > 6$        $x < -2$

The graph is the set of points greater than 6 (right of 6) or less than  $-2$  (left of  $-2$ ) but the endpoints are NOT included



### Example 8

Solve for  $x$   
 $|6x - 15| \geq 3$

greater than or = 3 or less than or =  $-3$   
 $6x - 15 \geq 3$  or  $6x - 15 \leq -3$   
 $6x - 15 \geq 3$        $6x - 15 \leq -3$   
  
 $6x \geq 18$        $6x \leq 12$   
  
 $x \geq 3$        $x \leq 2$

The graph is the set of points greater than or = to 3 (right of 3) or less than then or = 2 (left of 2) and the endpoints ARE included



**Case 2: If the absolute value is alone and is greater than a negative number or greater than or equal to a negative number then all real numbers are solutions to the equation.**

if  
 $|ax + b| > \text{negative number}$

than all numbers work

**Example 9**

Solve for x

$$|2x - 7| > -5$$

$|2x - 7|$  is ALWAYS POSITIVE  
so it is ALWAYS greater than  $-5$

All numbers work

if  
 $|ax + b| \geq \text{negative number}$

than all numbers work

**Example 10**

Solve for x

$$|-2x + 3| + 9 \geq 2$$

$$|-2x + 3| \geq -7$$

$|-2x + 3|$  is ALWAYS POSITIVE  
so it is ALWAYS greater than  $-7$

All numbers work