

Section 1 – 5: Solving Absolute Value Equations

Step 1: Get the absolute value expression **alone on one side of the equation**.

Step 2: If the absolute value expression **is alone** and is **equal to a negative number STOP**.
There is no solution. Write NS or use \emptyset the symbol.

Step 3: If the absolute value expression **is alone** and is **equal to a positive number then**
write 2 separate equations.

Solve for x:

$$|ax + b| = c \text{ (where } c \text{ is a positive number)}$$

write 2 separate equations

$$ax + b = c$$

or

$$ax + b = -c$$

set whats inside the brackets = c or set whats inside the brackets = $-c$

Example 1

Solve for x

$$|3x - 12| = 21$$

$$3x - 12 = 21 \quad \text{or} \quad 3x - 12 = -21$$

$$\begin{array}{r} 3x - 12 = 21 \\ +12 \quad +12 \end{array} \qquad \begin{array}{r} 3x - 12 = -21 \\ +12 \quad +12 \end{array}$$

$$3x = 33$$

$$3x = -9$$

$$x = 11$$

$$x = -3$$

Check: $|3x - 12| = 21$

Check: $|3x - 12| = 21$

$$|3(11) - 12| = 21$$

$$|3(-3) - 12| = 21$$

$$|33 - 12| = 21$$

$$|-9 - 12| = 21$$

$$|21| = 21$$

$$|-21| = 21$$

Example 2

Solve for x

$$|5x + 10| = 45$$

$$5x + 10 = 45 \quad \text{or} \quad 5x + 10 = -45$$

$$\begin{array}{r} 5x + 10 = 45 \\ -10 \quad -10 \end{array} \qquad \begin{array}{r} 5x + 10 = -45 \\ -10 \quad -10 \end{array}$$

$$5x = 35$$

$$5x = -55$$

$$x = 7$$

$$x = -11$$

Check: $|5x + 10| = 45$

Check: $|5x + 10| = 45$

$$|5(7) + 10| = 45$$

$$|5(-11) + 10| = 45$$

$$|35 + 10| = 45$$

$$|-55 + 10| = 45$$

$$|45| = 45$$

$$|-45| = 45$$

Get the absolute value expression **al alone on one side of the equation.**

Example 3

Solve for x

$$\begin{array}{r} |4x - 12| - 38 = -2 \\ + 38 \quad + 38 \end{array}$$

$$|4x - 12| = 36$$

$$-4x - 12 = 36 \quad \text{or} \quad -4x - 12 = -36$$

$$-4x - 12 = 36 \quad -4x - 12 = -36$$

$$-4x = 48 \quad -4x = -24$$

$$x = -12 \quad x = 6$$

Example 4

Solve for x

$$\begin{array}{r} |5x + 10| + 13 = 48 \\ -13 \quad -13 \end{array}$$

$$|5x + 10| = 35$$

$$5x + 10 = 35 \quad \text{or} \quad 5x + 10 = -35$$

$$\begin{array}{r} 5x + 10 = 35 \\ -10 \quad -10 \end{array} \quad \begin{array}{r} 5x + 10 = -35 \\ -10 \quad -10 \end{array}$$

$$5x = 25 \quad 5x = -45$$

$$x = 5 \quad x = -9$$

If the absolute value **is alone** and is **equal to a negative number** then the equation has **no solution.**

if

$$|ax + b| = \text{negative number}$$

then there is no solution

Example 5

Solve for x

$$|2x - 7| = -5$$

STOP

$|2x - 7|$ is ALWAYS POSITIVE
so it CANNOT equal -5

No Solution

Example 6
Get the absolute value alone

Solve for x

$$\begin{array}{r} |-2x + 3| + 9 = 2 \\ -9 \quad -9 \end{array}$$

$$|-2x + 3| = -7$$

STOP

$|-2x + 3|$ is ALWAYS POSITIVE
so it CANNOT equal -7

No Solution

**If the absolute value is alone and is equal to zero
then the equation has 2 identical solutions.**

if

$$|ax + b| = 0$$

then

$$ax + b = 0 \quad \text{or} \quad ax + b = -0$$

and both equations will have the same solution

Example 7

Solve for x

$$|2x - 6| = 0$$

$$2x - 6 = 0 \quad \text{or} \quad 2x - 6 = -0$$

$$2x - 6 = 0 \quad 2x - 6 = 0$$

$$2x = 6$$

$$2x = 6$$

$$x = 3$$

$$x = 3$$

Example 8

Solve for x

$$|-3x + 12| = 0$$

$$-3x + 12 = 0 \quad \text{or} \quad -3x + 12 = -0$$

$$-3x + 12 = 0 \quad -3x + 12 = 0$$

$$-3x = -12$$

$$-3x = -12$$

$$x = 4$$

$$x = 4$$

If each side of an equation has an absolute value that is alone

If

$$|ax + b| = |cx + e|$$

write 2 separate equations

$$ax + b = cx + e$$

set whats inside the left brackets = to whats inside the right brackets

$$ax + b = -(cx + e)$$

set whats inside the left brackets = -(whats inside the right brackets)

and solve both equations for x

Solve for x

$$|2x - 4| = |x - 8|$$

$$2x - 4 = x - 8 \quad \text{or} \quad 2x - 4 = -(x - 8)$$

$$2x - 4 = x - 8 \quad 2x - 4 = -(x - 8)$$

$$2x - 4 = -x + 8$$

$$x - 4 = -8$$

$$3x - 4 = 8$$

$$3x = 12$$

$$x = -4$$

$$x = 4$$

Solve for x:

$$|2x + 6| = |3x + 4|$$

$$2x + 6 = 3x + 4 \quad \text{or} \quad 2x + 6 = -(3x + 4)$$

$$2x + 6 = 3x + 4$$

$$2x + 6 = -(3x + 4)$$

$$2x + 6 = -3x - 4$$

$$+6 = x + 4$$

$$5x + 6 = -4$$

$$5x = -10$$

$$2 = x$$

$$x = -2$$