Section 1 – 4B: Compound Inequalities

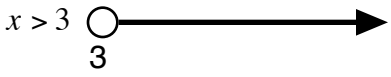
Some inequality statements contain **2 inequalities with an "and" or an "or" between them**. We call these type of inequalities **compound inequalities**.

$x > 3$ and $x < 5$ $\quad$ $x < 2$ and $8 \geq x$ $\quad$ $x \leq -4$ or $5 > x$ $\quad$ $x \geq 3$ or $5 \geq x$

Graphing the solution to a single Inequality

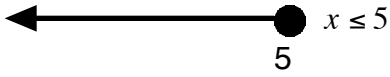
Graph : $x > 3$

When we graph the solution to a single inequality like $x > 3$ we shade in all the points that make this inequality true. These are all the points greater than 3



Graph : $x \leq 5$

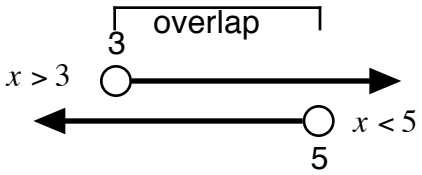
When we graph the solution to a single inequality like $x \leq 5$ we shade in all the points that make this inequality true. These are all the points less than or equal to 5



Graphing the solution to a compound Inequality with an "AND"

Graph the solution to $x > 3$ AND $x < 5$

The solution to $x > 3$ **AND** $x < 5$ **are all the points that will make both** $x > 3$ **AND** $x < 5$ true **at the same time.** To find these points we must look for the points that are on **BOTH** the $x > 3$ **graph AND the** $x < 5$ **graph.** If the graphs are graphed on the same line then **where the graphs overlap is the solution**. It is hard to see the overlap with both graphs on top of each other so we graph each separate graph **one above the other** and look for the **area where the graphs overlap**.

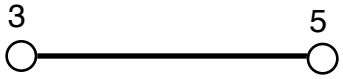


the points where the graphs overlap are
between 3 and 5
but the 3 and the 5 are NOT included.

The solution graph is the set of points
between 3 and 5
but the endpoints are **NOT** included.

Answer:

$3 < x < 5$



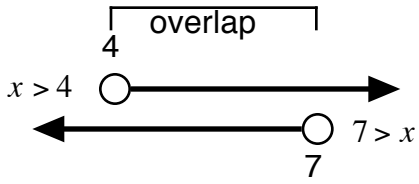
Interval Notation: x ∈ (3 , 5)

The "AND" Case

1. Graph each inequality on a separate number line one under the other.
2. Observe where the graphs **overlap.** Think of moving the graphs onto the same line and think where the shaded areas would touch. The area of overlap is the solution.

Example 1

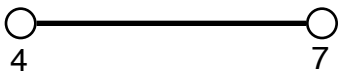
$x > 4$ and $7 > x$



the points where the graphs overlap are between 4 and 7
but the 4 and the 7 **are NOT** included

Answers:

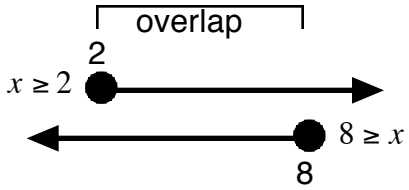
$4 < x < 7$



Interval Notation: x ∈ (4 , 7)

Example 2

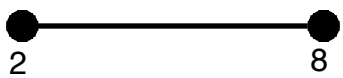
$x \geq 2$ and $8 \geq x$



The points where the graphs overlap are between 2 and 8
and the 2 and the 8 **ARE** included

Answers:

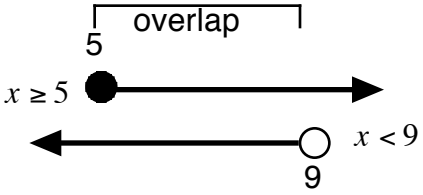
$2 \leq x \leq 8$



Interval Notation: x ∈[2 ,8]

Example 3

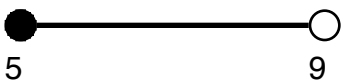
$x \geq 5$ and $x < 9$



the points where the graphs overlap are between 5 and 9
the 5 **IS** included but the 9 I**S NOT** included

Answers:

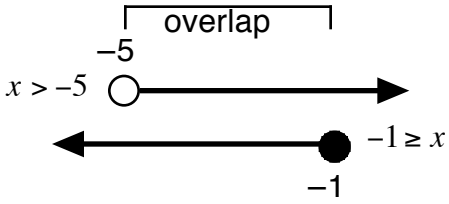
$5 \leq x < 9$



Interval Notation: x ∈ [5 , 9)

Example 4

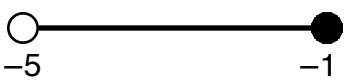
$x > -5$ and $-1 \geq x$



The points where the graphs overlap are between –5 and –1
the –5 **IS NOT** included but the –1 **IS** included

Answers:

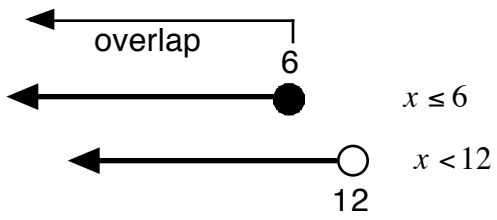
$-5 < x \leq -1$



Interval Notation: x ∈ (–5 , –1]

Example 5

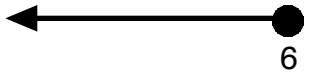
$x \le 6$ and $x < 12$



the points where the graphs overlap are
to the left of 6
and the 6 **IS** included

Answers:

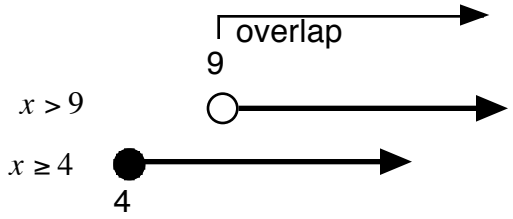
$x \le 6$



Interval Notation: x $\in$ **($-\infty$, 6]**

Example 6

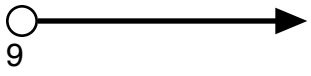
$x > 9$ and $x \ge 4$



The points where the graphs overlap are
to the right of 9
but the 9 is **NOT** included

Answers:

$x > 9$

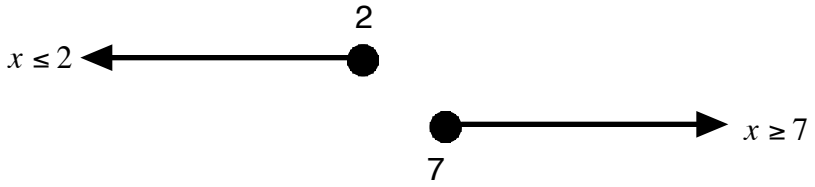


Interval Notation: x $\in$ **(9 , $+\infty$)**

Example 7

$x \le 2$ and $x \ge 7$

there is no overlap



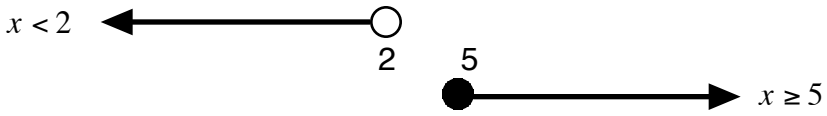
there are **NO POINTS** where the graphs overlap
so there is
No Solution

Graphing the Solution to Compound Inequality with an "OR"

Graph the solution to $x < 2$ or $x \geq 5$

The solution to $x < 2$ **OR** $x \geq 5$ **are all the points that will make** $x < 2$ **true combined with all the points that make** $x \geq 5$ **true.** The "OR" case requires we **take ALL THE POINTS on either of the separate graphs** instead of the overlap like we did with the "and" case. If the graphs were graphed on the same line we would **take any point that was shaded on any part of the graph**. We graph each separate graph one above the other and look for **any points on either graph**.

take all the points on either graph



we take the points to the left ot 2
combined with the
points to the right ot 5
and the 5 **IS** included.

Answers:

$x < 2$ or $x \geq 5$

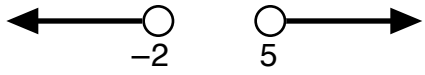
Interval Notation: $\mathbf{x} \in (-\infty\ , 2\ ) \cup [\ 5\ , +\infty\ )$

The "OR" Case

1. Graph each inequality on a separate number line one under the other.
2. Observe all the points where there are points that are shaded. Think of moving the graphs onto the same line and observe where the shaded areas. Any shaded area is part of the solution. **All the points on each of the separate graphs** are part of the solution.

Example 1

$x < -2$ or $x > 5$



The points to the **left of –2** are shaded
The points to the **right of 5** are shaded

All the points on each of the separate graphs are part of the solution

$\mathbf{x \in (-\infty, -2) \cup (5, +\infty)}$

Example 2

$x \le -4$ or $x \ge 8$

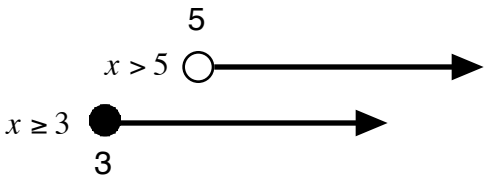
The points to the **left of 4** including 4 are shaded
The points to the **right of 8** including 8 are shaded

All the points on each of the separate graphs are part of the solution

$\mathbf{x \in (-\infty, -4] \cup [8, +\infty)}$

Example 3

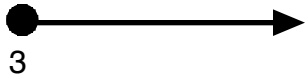
$x > 5$ or $x \ge 3$



the point 3 and all the points to the right of 3 are shaded by at least one of the graphs

Answers:

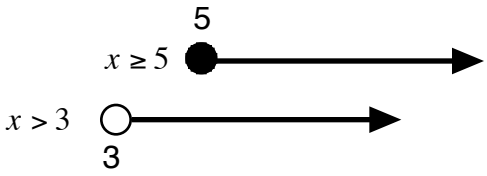
$x > 5$ or $x \ge 3$



$\mathbf{x \in [3, +\infty)}$

Example 4

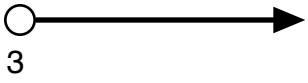
$x \ge 5$ or $x > 3$



the points to the right of 3 are shaded by at least one of the graphs

Answers:

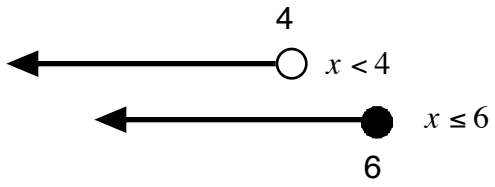
$x > 5$ or $x \ge 3$



$\mathbf{x \in (3, +\infty)}$

Example 5

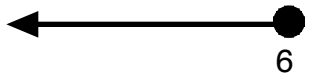
$x < 4$ or $x \le 6$



the point 6 and all the points to the left of 6
are shaded by at least one of the graphs

Answers:

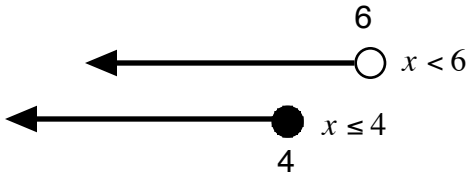
$x < 4$ or $x \le 6$



x $\in (-\infty , 6]$

Example 6

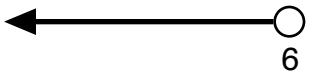
$x \le 4$ or $x < 6$



the points to the left of 6
are shaded by at least one of the graphs

Answers:

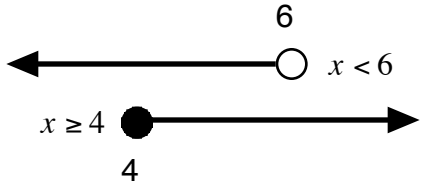
$x \le 4$ or $x < 6$



x $\in (-\infty , 6)$

Example 7

$x \ge 4$ or $x < 6$



All Points are shaded
by at least one of the graphs

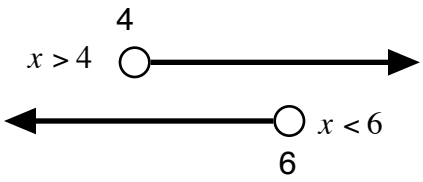
Answers:

All Real Numbers

x $\in (-\infty , +\infty)$

Example 8

$x > 4$ or $x < 6$



All Points are shaded
by at least one of the graphs

Answers:

All Real Numbers

x $\in (-\infty , +\infty)$

