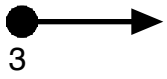
Section 1 – 4A: Interval Notation

The solution to an inequality represents all the values for x that make the inequality true. There are several ways to express these solutions:

English Sentence	Algebraic Inequality	Graph	Interval Notation
the values for x that make the inequality true are all **the real numbers** that are **greater than or equal to 3**	$x \geq 3$		$x \in [3, +\infty)$

The English Sentence Form

The English Sentence form states that **x must be a Real Number**. It also **states a range of the real numbers** that are solutions for the **variable x**. The sentence is easy to read but it does take up a lot of room. The other three forms shown above are preferred to the sentence because of their compact forms.

The Algebraic Inequality Form

The Algebraic Inequality form is the basis for almost every problem. Most problems start with a complex inequality and require you to solve for x. After you solve for x the solution will look like the simple Algebraic Inequality $x > 3$. Students like it because it is the most compact form but the direction of the symbol can confuse students. $x > 3$ and $3 < x$ both say **x is greater than three** but the version with x on the right $3 < x$ is often misread.

The Real Number Line graph Form

The graph form of the solution is helpful in many cases. There are an infinite number of real numbers that are solutions and a visual picture of all the solution can be helpful. The **number line you draw represents real numbers** and the **shaded part the line represents the Real Number solutions for x**. The arrow at the end of the graph indicates the direction and implies there is no end to the solutions. The values for the solution continues on to **infinity**. None of these facts are stated on the graph. They are understood assumptions.

The Interval Notation Form

This form is intend to combine the compact form of the algebraic inequality with an easy to read range to describe the solutions. The notation starts with stating that t**he variable is x.** **The $\in$ sign means "is an element of".** The $\in$ sign says that the solutions for x are all the numbers included in the set that follows the $\in$ sign. The set contains **the left and right limits for the range of solutions.** The (or [symbols are used as set brackets to indicate if the endpoints are included in the solution set or not. $-\infty$ and $+\infty$ symbols are used to indicate if the range of solutions continue on to **infinity**.

The Interval Notation Form
for stating Solutions to Inequalities

The format for Interval Notation **states the lowest and highest real numbers in the solution set** of an inequality and tells if the **endpoints are included** with use of **([ )]** symbols.

$3 < x < 5$	$3 \le x < 5$	$3 \le x \le 5$	$3 < x \le 5$
$x \in (3, 5)$	$x \in [3, 5)$	$x \in [3, 5]$	$x \in (3, 5]$

The **( )** Brackets are called "**soft**" and mean that **endpoints** of the set **ARE NOT INCLUDED**
The **[ ]** Brackets are called "**hard**" and mean that endpoints of the set are **ARE INCLUDED**

A $-\infty$ or $+\infty$ and a **Soft Bracket** indicate that the range of solutions continues to **infinity**.

$x < 5$	$x \le 5$	$x > 5$	$x \ge 5$
$x \in (-\infty, 5)$	$x \in (-\infty, 5]$	$x \in (5, +\infty)$	$x \in [5, +\infty)$

Solutions with two sets of solutions
The use of Union or U

Past sections have had single inequalities as a solution. This section will introduce solutions that have t**wo sets of solutions**. Interval notation requires that **both sets of solutions are listed** and the word OR is replaced with the symbol **U** for Union. The symbol **U** for Union means that all the real numbered solutions for x are found in **one OR the other of the two sets**.

$3 < x < 5$ or $x > 8$	$x \le 1$ or $x \ge 5$	$x \le 1$ or $3 \le x \le 5$
$x \in (3, 5) \cup (8, +\infty)$	$x \in (-\infty, 1] \cup [5, +\infty)$	$x \in (-\infty, 1] \cup [3, 5]$

A more formal format would also include the statement x is an element of $\in$ the Real Numbers $\Re$ at the start of the statement so it is very clear that the set of solutions for x are the real numbers.

$$x \in \Re \ \mathrm{I}\ x \in [5, +\infty)$$

is read:

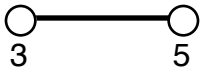
x is an element of the Real Numbers such that x is a member of the set [5 , $+\infty$)

at this point in the course we will assume that x is a real number so
we will use the abbreviated version $x \in [5, +\infty)$

Example 1

the solution set contains all the real numbers
between 3 and 5
but the 3 and the 5 are **NOT included.**

$3 < x < 5$



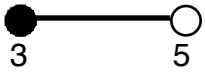
Interval Notation: x ∈ (3 , 5)

The (means the 3 is NOT included
The) means the 5 is NOT included

Example 2

the solution set contains all the real numbers
between 3 and 5
the 3 **IS included** and the 5 is **NOT included**

$3 \le x < 5$



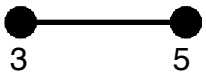
Interval Notation: x ∈ [3 , 5)

The [means the 3 is included
The) means the 5 is NOT included

Example 3

the solution set contains all the real numbers
between 3 and 5
but the 3 and the 5 are **NOT included.**

$3 \le x \le 5$



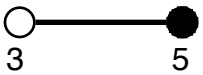
Interval Notation: x ∈ [3 , 5]

The [means the 3 is included
The] means the 5 is included

Example 4

the solution set contains all the real numbers
between 3 and 5
the 3 **IS included** and the 5 is NOT **included**

$3 < x \le 5$



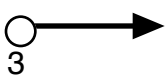
Interval Notation: x ∈ (3 , 5]

The (means the 3 is NOT included
The] means the 5 is included

Example 5

the solution set contains all the real numbers
from 3 to $+\infty$
but the 3 **IS NOT included.**

$x > 3$



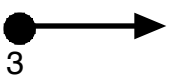
Interval Notation: x ∈ (3 , $+\infty$)

The (means the 3 is NOT included
A soft bracket) is used with $+\infty$

Example 6

the solution set contains all the real numbers
from 3 to $+\infty$
and the 3 **IS included.**

$x \ge 3$



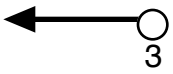
Interval Notation: x ∈ [3 , $+\infty$)

The [means the 3 is included
A soft bracket) is used with $+\infty$

Example 7

the solution set contains all the real numbers
from $-\infty$ to 3
but the 3 **IS NOT included.**

$x < 3$



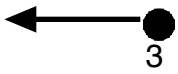
Interval Notation: x $\in (-\infty, 3)$

The) means the 3 is NOT included
A soft bracket (is used with $-\infty$

Example 8

the solution set contains all the real numbers
from $-\infty$ to 3
and the 3 **IS included.**

$x \leq 3$



Interval Notation: x $\in (-\infty, 3]$

The] means the 3 is included
A soft bracket (is used with $-\infty$

Example 9

the solution set contains all the real numbers
from $-\infty$ to 3 but the 3 **IS NOT included**
OR the real numbers
from 6 to $+\infty$ but the 6 **IS NOT included**

$x < 3$ or $x > 6$

x $\in (-\infty, 3) \cup (6, +\infty)$

The) means the 3 is NOT included
The) means the 6 is NOT included
A soft bracket (is used with $-\infty$
A soft bracket) is used with $+\infty$

Example 10

the solution set contains all the real numbers
from $-\infty$ to 3 and the 3 **IS included**
OR the real numbers
from 6 to $+\infty$ and the 6 **IS included**

$x \leq 3$ or $x \geq 6$

x $\in (-\infty, 3] \cup [6, +\infty)$

The] means the 3 IS included
The [means the 6 IS included
A soft bracket (is used with $-\infty$
A soft bracket) is used with $+\infty$

