

Section 1 – 3:

Solving Inequalities

Inequality Symbols

If two values are not equal to each other than one of the values must be the largest of the two values and the other value must be the smallest value. The inequality symbols **>** and **<** are used to show the relationship between two values that are not equal.

The larger value is on the side of the symbol with the **LARGE** opening and the smaller value is on the side with the **small pointed end**.

Larger value > Smaller value

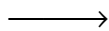
Smaller value < Larger value

One direction states the largest value first and the other direction states the smallest value first. When the larger value is read first the greater than phrase is used and when the smaller value is read first the less than phrase is used.

It is a common mistake to call **>** a greater than symbol

8 > 3 can be read from left to right as well as right to left.

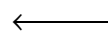
$$8 > 3$$



read left to right

eight is greater than 3

$$8 > 3$$



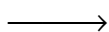
read right to left

3 is less than 8

It is also a common mistake to call **<** a less than symbol.

5 < 9 can be read from left to right as well as right to left.

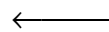
$$5 < 9$$



read left to right

five is less than 9

$$5 < 9$$



read right to left

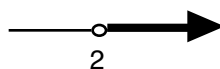
9 is greater than 5

Inequalities with an unknown variable

An inequality with a variable like $x > 2$ is read x is greater than 2 and is used to express all the numbers greater than 2. There are many numbers greater than 2.

2.000001 , 2.001 , 2.01 , 2.9 , 3 , 4 , 6 , 9 , 119 and on and on are all numbers greater than 2

A number line is used to show all the numbers that are solutions to $x > 2$. To show all of the numbers greater than 2 draw a number line with a 2 under it. An open circle above the 2 is used to show that 2 is not part of the solution as 2 is not greater than 2. Then all the numbers greater than 2 are shaded in. An arrow is used to show that the solutions go on in that direction without end.



Reading and Graphing Inequalities

Inequalities are statements with expressions related by one of the four inequality symbols

$$> , \geq , < , \leq$$

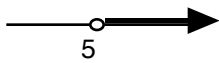
Read The Variable Side First

DO NOT read every inequality from left to right

Read each inequality by **reading the x term first** no matter which side it is on
and then continue reading in the direction needed to complete the sentence.

Example 1

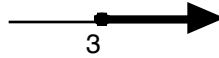
The inequality $x > 5$ is read
 x is greater than 5
and the graph is



The open circle means
5 is not a solution
but all real numbers to the
right of 5 are solutions

Example 2

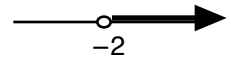
The inequality $3 \leq x$ is read
 x is greater than or equal to 3
and the graph is



The closed circle means
3 is a solution
and all real numbers to the
right of 3 are also solutions

Example 3

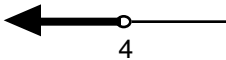
The inequality $x > -2$ is read
 x is greater than -2
and the graph is



The open circle means
 -2 is not a solution
but all real numbers to the
right of -2 are solutions

Example 4

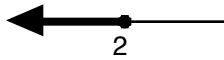
The inequality $x < 4$ is read
 x is less than 4
and the graph is



The open circle means
4 is not a solution
but all real numbers to the
left of 4 are solutions

Example 5

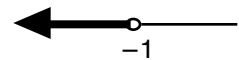
The inequality $2 \geq x$ is read
 x is less than or equal to 2
and the graph is



The closed circle means
2 is a solution
and all real numbers to the
left of 2 are also solutions

Example 6

The inequality $-1 > x$ is read
 x is less than -1
and the graph is



The open circle means
 -1 is not a solution
but all real numbers to the
left of -1 are solutions

Solving Inequalities

1. **Distribute and combine like terms** on each side of the equal sign.
2. **Get the x term on only one side of the equation** by adding or subtracting the smallest x term from both sides of the inequality.
3. **Eliminate the constant term on the x side of the equation** by adding or subtracting that constant from both sides of the inequality.
5. **Eliminate the coefficient of the x term** by multiplying or dividing both sides of the inequality by the coefficient. **There is an additional part to this step if you divide both sides of the inequality by a negative number:**

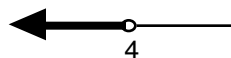
If you multiply or divide both sides of the inequality by a negative number then you must switch the direction of the inequality symbol.

Example 1

$$\begin{array}{r} 3x - 4 < 8 \\ +4 \quad +4 \end{array}$$

$$3x < 12$$

$$\begin{array}{r} \frac{3x}{3} < \frac{12}{3} \\ \mathbf{x < 4} \end{array}$$

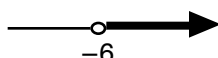


Example 2

$$\begin{array}{r} 22 > -3x + 4 \\ -4 \quad -4 \end{array}$$

$$18 > -3x$$

$$\begin{array}{r} \frac{18}{-3} < \frac{-3x}{-3} \\ \mathbf{-6 < x} \end{array}$$

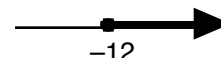


Example 3

$$\begin{array}{r} \frac{x}{-3} - 5 \leq -1 \\ +5 \quad +5 \end{array}$$

$$\frac{x}{-3} \leq 4$$

$$\begin{array}{r} (-3)\frac{x}{-3} \geq 4(-3) \\ \mathbf{x \geq -12} \end{array}$$

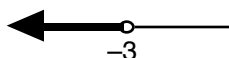


Example 4

$$\begin{array}{r} -5x + 6 > -x + 22 \\ +5x \quad +5x \end{array}$$

$$\begin{array}{r} 10 > 4x + 22 \\ -22 \quad -22 \end{array}$$

$$\begin{array}{r} -12 > 4x \\ \frac{-12}{4} > \frac{4x}{4} \\ \mathbf{-3 > x} \end{array}$$

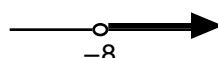


Example 5

$$\begin{array}{r} x + 9 > -2x - 15 \\ +2x \quad +2x \end{array}$$

$$\begin{array}{r} 3x + 9 > -15 \\ -9 \quad -9 \end{array}$$

$$\begin{array}{r} 3x > -24 \\ \frac{3x}{3} > \frac{-24}{3} \\ \mathbf{x > -8} \end{array}$$



Example 6

$$-2(x - 3) \leq -8$$

$$\begin{array}{r} -2x + 6 \leq -8 \\ -6 \quad -6 \end{array}$$

$$-2x \leq -14$$

$$\begin{array}{r} \frac{-2x}{-2} \geq \frac{-14}{-2} \\ \mathbf{x \geq 7} \end{array}$$

