

Section 1 – 1B:

Advanced Word Problems

Wet Mixture Examples

A solution is made by having a solid dissolved in a liquid (normally water).

The percent of the solution is based on the amount of the solid dissolved in the water.

A 20% salt water solution means that 20% of the total weight of the solution is salt and 80% of the weight is water.

A 30% acid solution means that 30% of the total weight of the solution is an ionic salt that will create the acid when put into water and 70% of the weight of the solution is water.

Pure water has no solid dissolved in it so it is a 0% solution based on the percent solid.

To create a final mixture the solution in Beaker 1 and the solution in beaker 2 are put into a third beaker and mixed.

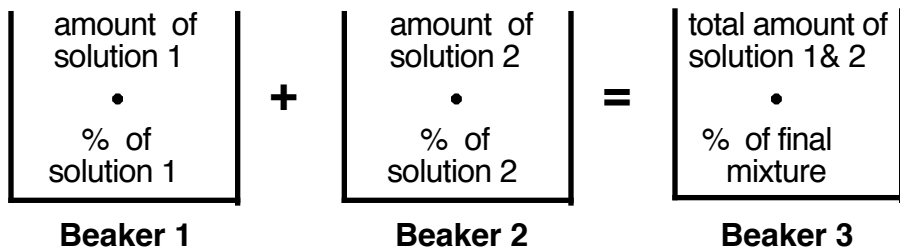
1. We will assume that the volume of the final mixture in Beaker 3 is the total volume of Beaker 1 and Beaker 2. Our mixtures are water based so this is a solid assumption.

volume in Beaker 1 + volume in Beaker 2 = total volume of the final mixture

another way of saying this is

total volume of the final mixture – volume in one beaker = volume in the other beaker

2. The % solution in beaker 3 will be given. The weaker % solution dilutes the stronger % solution to create a new mixture with a % solution somewhere between the % in Beaker 1 and the % in Beaker 2.



The equation you will use is shown below

$$\left(\begin{array}{c} \text{amount of} \\ \text{solution 1} \end{array} \right) \cdot \left(\begin{array}{c} \% \text{ of} \\ \text{solution 1} \end{array} \right) + \left(\begin{array}{c} \text{amount of} \\ \text{solution 2} \end{array} \right) \cdot \left(\begin{array}{c} \% \text{ of} \\ \text{solution 2} \end{array} \right) = \left(\begin{array}{c} \text{total amount of} \\ \text{solution 1 \& 2} \end{array} \right) \cdot \left(\begin{array}{c} \% \text{ of final} \\ \text{mixture} \end{array} \right)$$

Wet Mixture Example 1

Ann Marie wants to mix **some** 80% acid mixture with 230 ml of a 20% acid mixture to produce a 60% acid mixture. How many ml of the 80% acid mixture must be used?

Remember: amount of 1st mixture + amount of 2nd mixture = Total amount

$$\begin{array}{|c|} \hline x \\ \hline 80\% \\ \hline \end{array} + \begin{array}{|c|} \hline 230 \\ \hline 20\% \\ \hline \end{array} = \begin{array}{|c|} \hline x + 230 \\ \hline 60\% \\ \hline \end{array}$$

$$\left(\begin{array}{c} \text{amount of} \\ \text{solution 1} \end{array} \right) \cdot \left(\begin{array}{c} \% \text{ of} \\ \text{solution 1} \end{array} \right) + \left(\begin{array}{c} \text{amount of} \\ \text{solution 2} \end{array} \right) \cdot \left(\begin{array}{c} \% \text{ of} \\ \text{solution 2} \end{array} \right) = \left(\begin{array}{c} \text{total amount of} \\ \text{solution 1 \& 2} \end{array} \right) \cdot \left(\begin{array}{c} \% \text{ of final} \\ \text{mixture} \end{array} \right)$$

Amount of 80% solution = x

Amount of 20% solution = 230

$$x(.80) + 230(.20) = (x + 230)(.60) \quad \text{multiply each term by 100 to eliminate decimals}$$

$$x(80) + (230)(20) = (x + 230)(60)$$

$$80x + 4600 = 60x + 13800$$

$$20x = 9200$$

$$\mathbf{x = 460}$$

Answer: x = 460 ml of the 80% acid mixture

$$\text{Check: } 460(.80) + 230(.20) = (460 + 230)(.60)$$

Wet Mixture Example 2:

David wants to mix some 70% acid mixture with some ml of a 30% acid mixture to produce 400 ml of a 40% acid mixture. How many ml of the 70% acid and the 30% acid are needed?

total amount of the final mixture – amount in one beaker = amount in the other beaker

$$\begin{array}{|c|} \hline x \\ \hline 70\% \\ \hline \end{array} + \begin{array}{|c|} \hline 400 - x \\ \hline 30\% \\ \hline \end{array} = \begin{array}{|c|} \hline 400 \\ \hline 40\% \\ \hline \end{array}$$

$$\left(\begin{array}{c} \text{amount of} \\ \text{solution 1} \end{array} \right) \cdot \left(\begin{array}{c} \% \text{ of} \\ \text{solution 1} \end{array} \right) + \left(\begin{array}{c} \text{amount of} \\ \text{solution 2} \end{array} \right) \cdot \left(\begin{array}{c} \% \text{ of} \\ \text{solution 2} \end{array} \right) = \left(\begin{array}{c} \text{total amount of} \\ \text{solution 1 \& 2} \end{array} \right) \cdot \left(\begin{array}{c} \% \text{ of final} \\ \text{mixture} \end{array} \right)$$

Amount of 70% solution = x

Total amount of the final mixture = 400

Amount in 30% solution = 400 – x

$$x(.70) + (400 - x)(.30) = (400)(.40) \quad \text{multiply each term by 100 to eliminate decimals}$$

$$x(70) + (400 - x)(30) = (400)(40)$$

$$70x + 12000 - 30x = 16000$$

$$40x = 4000$$

$$x = 100$$

Answer: 100 ml of the 70% acid mixture and 300 ml of the 30% acid mixture

$$\text{Check: } 100(.70) + 300(.30) = (400)(.40)$$

Age Problems Introduction

Years Ago Example 1

David's "is" 7 years older than Tom. Five years ago David was twice as old as Tom. Find their ages now.

Age problems start by representing the ages of two different people **at the present time**. The "is" in the sentence means that the sentence is referring to their **present ages**.

Use the "is" clue to express their ages in the present (now)

The first sentence says: **David's "is" 7 years older than Tom.**

$$\text{Tom's age now} = x$$

$$\text{David's age now} = x + 7 \quad (\text{seven year older})$$

The second sentence says "**5 years ago**". The ages in this sentence are based on the past **when each person was 5 years younger than they are now**. You **subtract 5 from each of their current ages**. If Tom is x now he was $x - 5$ then. If David is $x + 7$ now he was $x + 7 - 5$ then.

	age now "is"	age 5 years ago each one is 5 years younger
Tom	x	$x - 5$
David	$x + 7$	$x + 7 - 5 = x + 2$

Now relate the **expressions for 5 years ago based on the second sentence**.

Five years ago

David **was** twice as old as Tom

$$\text{David} = 2 (\text{Tom})$$

substitute the expressions for David's and Tom's age **based on 5 years ago**

$$\text{David} = 2 (\text{Tom})$$

$$x + 2 = 2(x - 5)$$

$$x - 5 = 2x - 10$$

$$12 = x$$

Answer: Tom is now 12 years old and David is 19 years old

Check: Five years ago Tom was 7 and David was 14. David was twice as old as Tom then.

Years Ago Example 2

Julie “is” three times as old as Ann Marie. Six years ago Julie was 24 years older than Ann Marie. Find their ages now.

Use the “is” clue to express their ages in the present (now)

The first sentence says: Julie “is” three times as old as Ann Marie.

Ann Marie’s age now = x

Julie’s age now = $3x$ (three times older)

The second sentence says “6 years ago”. The ages in this sentence are based on the past when each person was 6 years younger than they are now. You subtract 6 from each of their current ages. If Ann Marie is x now she was $x - 6$ then. If Julie is $3x$ now she was $3x - 6$ then.

	age now “is”	age 6 years ago each one is 6 years younger
Ann Marie	x	$x - 6$
Julie	$3x$	$3x - 6$

Now relate the expressions for 6 years ago based on the second sentence.

six years ago

Julie was 24 years older than Ann Marie

you need to add 24 to Ann Marie's age to make her equal to Julie's age

$$\text{Julie} = \text{Ann Marie} + 24$$

substitute the expressions for Ann Marie and Julie's age based on 6 years ago

$$\text{Julie} = \text{Ann Marie} + 24$$

$$3x - 6 = (x - 6) + 24$$

$$3x - 6 = x + 18$$

$$12 = x$$

Answer: Ann Marie is now 12 years old and Julie is 36 years old

Check: Six years ago Ann Marie was 6 and Julie was 30. Julie was 24 years older than Ann Marie

Years From Now Example 1

Tom “is” seven years older than John. Two years from now Tom will be twice as old as John. Find their ages now.

Use the “is” clue to express their ages in the present (now)

The first sentence says: Tom “is” seven years older than John.

$$\text{John's age now} = x$$

$$\text{Tom's age now} = x + 7 \quad (\text{seven years older})$$

The second sentence says “2 years from now”. The ages in this sentence are based on the future when each person will be two years older than they are now. You add 2 to each of their current ages. If John is x now he will be $x + 2$ then. If Tom is $x + 7$ now he will be $x + 7 + 2$ then.

	age now “is”	age 2 years from now each one is 2 years older
John	x	$x + 2$
Tom	$x + 7$	$x + 7 + 2 = x + 9$

Now relate the expressions for 2 years from now based on the second sentence.

Two years from now

Tom will be twice as old as John.

$$\text{Tom} = 2 (\text{John})$$

substitute the expressions for Tom and John's age based on 2 years from now

$$\text{Tom} = 2 (\text{John})$$

$$x + 9 = 2(x+2)$$

$$x + 9 = 2x + 4$$

$$5 = x$$

Answer: John is now 5 years old and Tom is 12 years old

Check: Two years from now John will be 7 and Tom will be 14. Tom will be twice as old as John

Years From Now Example 2

Julie “is” five years younger than Joe. 7 years from now their ages will total 71. Find their ages now.

Use the “is” clue to express their ages in the present (now)

The first sentence says: Julie “is” five years younger than Joe

$$\text{Joe's age now} = x$$

$$\text{Julie's age now} = x - 5 \quad (\text{five years younger})$$

The second sentence says “7 years from now”. The ages in this sentence are based on the future when each person will be 7 years older than they are now. You add 7 to each of their current ages. If Joe is x now he will be $x + 7$ then. If Julie is $x - 5$ now she will be $x - 5 + 7$ then.

	age now “is”	age 7 year form now each one is 2 years older
Joe	x	$x + 7$
Julie	$x - 5$	$x - 5 + 7 = x + 2$

Now relate the expressions for 7 years form now based on the second sentence.

Seven years from now

Their ages will total 71

$$\text{Joe} + \text{Julie} = 71$$

substitute the expressions for Tom and John’s age based on 7 years form now

$$\text{Joe} + \text{Julie} = 71$$

$$x + 7 + x + 2 = 71$$

$$2x + 9 = 71$$

$$2x = 62$$

$$x = 31$$

Answer: Joe is now 31 years old and Julie is 26 years old

Check: Seven years from now Joe will be 38 and Julie will be 33. Their ages will total 71.

Simple Average Example

If **all the tests have the same point total** then
The average score for N tests is found using the following formula

$$\text{Average score on } n \text{ tests} = \frac{\text{test 1} + \text{test 2} + \text{test 3} + \text{test 4} + \dots + \text{test N}}{N}$$

Julie wants to have an average score of 80 on all five of the 100 point tests in her math class. She has scored 72, 89, 91, and 85 on her first four exams. What must she score on the last exam to achieve her goal?

Let: test 1 = 72 test 2 = 89 test 3 = 91 test 4 = 85 test 5 = x

$$\text{Avg. test score} = \frac{\text{test 1} + \text{test 2} + \text{test 3} + \text{test 4} + \dots + \text{test N}}{N}$$

$$80 = \frac{72 + 89 + 91 + 85 + x}{5}$$

$$400 = 337 + x$$

$$63 = x$$

Answer: Julie needs to score 63 points on her last test.

$$\text{Check: } \frac{72 + 89 + 91 + 85 + 63}{5} = 80$$

Weighed Averages

The formula above required that **all the tests have the same point total**. If some tests are worth 100 points and others are worth 50 points that formula will not work. The 100 point exam carries more weight in the final grade than the 50 point test.

You cannot find the average value of several items that have different values. If the worth of each item varies then the different values or weights for each item must be accounted for. We call the average value of the several items of different value the **Weighed Average**. The formula below finds the average value for several items where the values or weights of each item is not the same.

$$\text{Weighed Average} = \left(\text{value of item}_1 \right) \left(\frac{\% \text{ of total}}{\text{for item}_1} \right) + \left(\text{value of item}_2 \right) \left(\frac{\% \text{ of total}}{\text{for item}_2} \right) + \left(\text{value of item}_3 \right) \left(\frac{\% \text{ of total}}{\text{for item}_3} \right) + \dots$$

Weighed Average Example 1

Thomas wants to get a average score of 92 points in his math class. **Tests are worth 50%** of the total grade, **homework counts 30%** of the total grade and the final exam counts 20% of the total grade. Tom has a test average of 88 and a homework average of 97. What does Tom have to get on the final to achieve his goal?

% of total for tests = 50% % of total for homework = 30% % of total for homework = 20%

test average = 88

homework average = 97

final exam score = x

$$\text{Weighed Average} = \left(\text{value of item}_1 \right) \left(\frac{\% \text{ of total}}{\text{for item}_1} \right) + \left(\text{value of item}_2 \right) \left(\frac{\% \text{ of total}}{\text{for item}_2} \right) + \left(\text{value of item}_3 \right) \left(\frac{\% \text{ of total}}{\text{for item}_3} \right) + \dots$$

$$92 = 88(.50) + 97(.30) + x(.20)$$

$$92 = 44 + 29.1 + .2x$$

$$18.9 = .2x$$

$$94.5 = x$$

Answer: Thomas needs to get at least 94.5 points on the final