Section 1 – 1A: Basic Word Problems

Coin Problems

Coin problems involve finding **the number coins of different values** when **given the total value for all the coins.**

We will start to develop an equation we will use to solve these type of problems by looking at an example. If Mary had **12 dimes** and **20 nickels** an **expression for the total value** of the coins could be found by

multiply the **number of dimes (12) by the value for 1 dime (.10)** or 12(.10) = 1.20
and
multiply the **number of nickels (20) times the value for 1 nickel (.05)** or 20 (.05) = 1.00

and adding these two products together to get **the total value of the coins**

the final equation would look like this

12(.10) + 20 (.05) = $ 2.20

Note: we list the value of the coins as a decimal number with two decimal places.

Nickels = .05 **Dimes** = .10 **Quarters** = .25

In coin problems you will **KNOW** the **value of each coin** and the **total value of all the coins**. You will then be asked to find how many of each type of coin you need to give you the total value of all the coins.

A key sentence will contain an **expression** for the number of **one type of coins in terms of the other type of coin** (ie. There are **3 more dimes than nickels**) You will use that sentence to **express the number of one type of coin as x. The other type of coin will then be expressed as an algebraic expression in terms of x.**

The equation you will use is the same one you used above

$$\begin{pmatrix}\text{number of 1}\\ \text{type of coin}\end{pmatrix}\begin{pmatrix}\text{value of 1 coin of}\\ \text{that type}\end{pmatrix}+\begin{pmatrix}\text{number of the other}\\ \text{type of coin}\end{pmatrix}\begin{pmatrix}\text{value of 1 coin of}\\ \text{that type}\end{pmatrix}=\begin{matrix}\text{total value of}\\ \text{all the coins}\end{matrix}$$

Note : The value of money is shown as a decimal number with two decimal places. After we get the algebraic equation **multiply each term by 100** (**move the decimal point for each term 2 places to the right**) to produce an equivalent equation without decimals.

Coin Problem Example 1:

Tom has **4 more then three times as many dimes than nickels.** If the t**otal value of all the coins he has is $ 2.15** then find how many of each type of coin he has.

Let the number of nickels = x

Let the number of dimes = 3x + 4 (**4 more then three times as many dimes than nickels**)

$$\begin{pmatrix}\text{number of}\\ \text{nickels}\end{pmatrix}\begin{pmatrix}\text{value of}\\ \text{a nickel}\end{pmatrix}+\begin{pmatrix}\text{number of}\\ \text{dimes}\end{pmatrix}\begin{pmatrix}\text{value of}\\ \text{a dime}\end{pmatrix}=\begin{matrix}\text{total value of}\\ \text{all the coins}\end{matrix}$$

$$(x)\quad(.05)\quad+\quad(3x+4)\quad(.10)\quad=2.15$$

nickels **dimes**

x(.05) + (**3x + 4**)(.10) = 2.15

x(5) + (3x + 4)(10) = 215 (multiply each term by 100 to eliminate decimals)

5x + 30x + 40 = 215 distribute the 10

35x + 40 = 215

35x = 175

x = 5 (the number of **nickels**)

and the number of **dimes** = 3x + 4 = 3(5) + 4 = 15 + 4 = 19

Answer: The number of nickels is 5 and the number of dimes is 19

Check: 5(.05) + 19(.10)
= .25 + 1.90
= 2.15

Coins Problem Example 2:

Mary has **4 less than twice as many quarters than she has** dimes. The total value of the coins is **\$ 6.20.** Find how many of each type of coin she has.

Let the number of dimes = x

Let the number of quarters = 2x – 4 (**4 less than twice as many** quarters as dimes)

$$\begin{pmatrix}\text{number of}\\ \text{dimes}\end{pmatrix}\begin{pmatrix}\text{value of}\\ \text{a dime}\end{pmatrix}+\begin{pmatrix}\text{number of}\\ \text{quarters}\end{pmatrix}\begin{pmatrix}\text{value of}\\ \text{a quarter}\end{pmatrix}=\begin{matrix}\text{total value of}\\ \text{all the coins}\end{matrix}$$

$$(x) \quad (.10) \quad + \quad (2x-4) \quad (.25) \quad = 6.20$$

dimes **quarters**

x (.10) + (**2x – 4**) (.25) = 6.20

x(.10) + (2x – 4)(.25) = 6.20 (multiply each term by 100 to eliminate decimals)

x(10) + (2x – 4)(25) = 620 distribute the 25

10x + 50x – 100 = 620

60x – 100 = 620

60x = 720

x = 12 (the number of **dimes**)

and the number of **quarters** = 2x – 4 = 2(12) – 4 = 20

Answer: The number of dimes is 12 and the number of quarters is 20

Check: 12(.10) + 20(.25) = 1.20 + 5.00 = \$ 6.20

Sales Problem Examples

Sales Problems are very similar to the coin problems above. There are 2 (or more) items to sell and each item has a different selling price. You will **KNOW** the **selling price of each of the different items** and the **total receipts from selling all the items**. You will then be asked to find the number of each of the different items that were sold.

A key sentence will contain an **expression** for the number of of the items sold in terms of a different item to be sold. (ie. Sue sold **3 more jackets than shirts**) You will use that sentence to **express one of the items as x. The other type of item will then be expressed as an algebraic expression in terms of x.**

The equation you will use is shown below

$$\begin{pmatrix}\text{the number sold}\\ \text{of one item}\end{pmatrix}\begin{pmatrix}\text{selling price of}\\ \text{1 item that type}\end{pmatrix}+\begin{pmatrix}\text{the number sold}\\ \text{of a second item}\end{pmatrix}\begin{pmatrix}\text{selling price of}\\ \text{1 item that type}\end{pmatrix}=\text{total receipts}$$

Sales Problem Example 1:

At a back to school event at the local preschool, teachers are going to sell sodas and hot dogs. A can of **soda sells for 20 cents** and a **hot dog sells for 60 cents**. After the event was over they found that they **had** sold **6 more hot dogs than soda pops**. How many hot dogs and soda pops did they sell if the total receipts were $9.20?

Let the number of soda pop tickets = x

Let the number of hot dog tickets = x + 6 (Note: 6 more hot dog tickets as soda pop tickets)

$$\begin{pmatrix}\text{the number of}\\ \text{soda pop sold}\end{pmatrix}\begin{pmatrix}\text{selling price of}\\ \text{1 soda pop}\end{pmatrix}+\begin{pmatrix}\text{the number of}\\ \text{hot dogs sold}\end{pmatrix}\begin{pmatrix}\text{selling price of}\\ \text{1hot dog}\end{pmatrix}=\text{total receipts}$$

$$(x) \qquad (.20) \qquad + \qquad (x+6) \qquad (.60) \qquad = 9.20$$

soda pop **hot dog** **Total**

x (.20) + (x + 6) (.60) = 9.20

x (.20) + (x + 6) (.60) = 9.20 (multiply each term by 100 to eliminate decimals)

x(20) + (x + 6)(60) = 920 (distribute the 60)

20x + 60x + 360 = 920

80x = 560

x = 7 soda pop tickets

and the number of hot dog tickets = x + 6 = 7 + 6 = 13

Answer:

The number soda pops sold was 7 and the number of hot dog sold was 13

Check: 7(.20) + 13(.60)
= 1.40 + 7.80
= $ 9.20

Sales Problem Example 2:

Dimple Records sells used CD's for $ 2.50 each and new Cd's for $ 8.00 each. One day the number of **new CD's** sold was **8 less than twice as many used CD's**. If the total sells were $ 121 , how many of each type did they sell?

Let the number of **used CD's** = x

Let the number of **new CD's** = 2x – 8 Note: new was 8 less than twice as many used CD's

$$\begin{pmatrix}\text{the number of}\\ \text{used cd's sold}\end{pmatrix}\begin{pmatrix}\text{selling price of}\\ \text{1 used CD}\end{pmatrix}+\begin{pmatrix}\text{the number of}\\ \text{new CD's sold}\end{pmatrix}\begin{pmatrix}\text{selling price of}\\ \text{1new CD}\end{pmatrix}=\text{total receipts}$$

$$(x) \qquad (2.50) \qquad + \qquad (x+8) \qquad (8.00) \qquad =121.00$$

2.50 CD **8.00 CD**

x (2.50) + (2x – 8) (8.00) = 121.00

x (2.50) + (2x – 8)(8.00) =121.00 (multiply each term by 100)

x (250) + (2x – 8)(800) = 12100 (distribute the 800)

250 x + 1600x – 6400 = 12100 (combine the x terms)

1850 x – 6400 = 12100

1850 x = 18500

x = 10 used CD's

and the number of **new CD's** is 2x – 8 = 2(10) – 8 = 20 – 8 = 12

Answer: The number of used CD's is 10 and the number of new CD's is 12

Check: 10(.2.50) + 12(.8.00)
= 25 + 96
= $ 121

Investment Examples

Investment Problems are very similar to the Sales problems above. There are 2 (or more) investment accounts and each account pays a **different rate of simple annual interest**. You will **KNOW** the rate of simple annual interest for each of the different accounts and the **total annual interest**. You will then need to find the **amount to invest in each of the accounts** so that the investments will return the total annual interest stated in the problem.

A key sentence will contain an **expression** for the amount of many in one account in terms of the amount of money in the other accounts. (ie. Julie invested twice as much at 5% as she did at 8%). You will use that sentence to **express the amount of money in one of the accounts as as x.** The **amount of money in the other account** will then be expressed as an **algebraic expression in terms of x.**

The equation you will use is shown below

$$\begin{pmatrix}\text{amount of \$}\\ \text{in one account}\end{pmatrix}\begin{pmatrix}\text{\% interest}\\ \text{for that account}\end{pmatrix}+\begin{pmatrix}\text{amount of \$}\\ \text{in 2nd account}\end{pmatrix}\begin{pmatrix}\text{\% interest}\\ \text{for that account}\end{pmatrix}=\begin{pmatrix}\text{total annual}\\ \text{interest earned}\end{pmatrix}$$

Investment Example 1:

Ann Marie wants to invest **some** of her in a CD t**hat pays 8%** interest and **twice as much** in a CD that **pays 10%**. She wants to **earn $ 1400** interest in one year. How much money should she put in each account?

Let the amount at 8% = x

Let the amount at 10% = 2x (**twice as much**)

$$\begin{pmatrix}\text{amount of \$}\\ \text{in one account}\end{pmatrix}\begin{pmatrix}\text{\% interest}\\ \text{for that account}\end{pmatrix}+\begin{pmatrix}\text{amount of \$}\\ \text{in 2nd account}\end{pmatrix}\begin{pmatrix}\text{\% interest}\\ \text{for that account}\end{pmatrix}=\begin{pmatrix}\text{total annual}\\ \text{interest earned}\end{pmatrix}$$

8% account   **10% account**

x (.08) + (2x) (.10) = 1400.00   (some at 8% and twice as much at 10%)

x(.08) + 2x(.10) = 1400.00   (multiply each term by 100)

x(8) + 2x(10) = 140,000   multiply

8x + 20 = 140,000

28x = 140,000

x = 5,000

Answer: $ 5,000 at 8% and **$ 10,000 at 10%**

Investment Example 2:

Only the total invested in both accounts is given

NOTE: If you are given a **total amount** to invest in **two accounts** then
the amount of money in one account is x
and the amount of money in the other account is Total – x

David wants to invest **some** of his savings in a bank **that pays 5%** interest and **the rest of his money** in a CD **that pays 8%**. He has a total of **$ 2000 to invest.** How much money should he put in the two accounts if he wants to earn **$ 124 total annual interest?**

It does not make a difference which of the two accounts is labeled x. The other one will be Total – x

Let the **amount at 5% = x**

Let the amount at 8% = 2000 – x

$$\begin{pmatrix}\text{amount of}\\ \$ \text{ at } 5\%\end{pmatrix}\begin{pmatrix}\%\text{ interest}\\ \text{for that account}\end{pmatrix}+\begin{pmatrix}\text{the rest of the}\\ \$ \text{ 2000\$ at } 8\%\end{pmatrix}\begin{pmatrix}\%\text{ interest}\\ \text{for that account}\end{pmatrix}=\begin{pmatrix}\text{total annual}\\ \text{interest earned}\end{pmatrix}$$

$$x \quad (.05) \quad + \quad (2000-x) \quad (.08) \quad = \quad 124.00$$

5% account **8% account**

x (.05) + (**2000 – x**) (.08) = 124.00 (some at 5% and the rest at at 8%)

x (.05) + (2000 – x) (.08) = 124.00 (multiply each term by 100)

x (5) + (2000 – x) (8) = 12400 (Distribute the 8)

5x + 16000 – 8x = 12400

–3x + 16000 = 12400

–3x = –3600

x = 1200

Answer: $1200 at 5% and $800 at 8%

Check: 1200 (.05) + 800 (.08) = 60 + 64 = 124

