Section 0 – 7: Solving Linear Equations

The difference between an expression and an equation

Expressions do not contain an equal sign. An **expression can be simplified to get a new expression. Equations contain an equal sign.** The properties of equality are used to help solve equations. They **CANNOT** be used on expressions.

Expressions can be simplified

$3(x-5)-2(x+1)$

is simplified to

$3x-15+2x+2$

and then simplified further to

$5x-13$

Equations can be solved

$x-3=2$

can be solved by

addinga 3 to both

sides of the equation

$x=5$

The Solution to an Equation

Equations can be solved. The solution to an equation is **a value for x that can be substituted into the equation for the x** variable and the resulting equation must be **TRUE.**

Substitute the given value of x for the variable and determine if it is a solution

Example 1

Is

$x=-3$

a solution to the equation

$-4x+1=13$

replace x with -3 in

$-4x+1=13$

this yeilds

$-4(-3)+1=13$

which reduces to

$13=13$

so $x=-3$

IS A SOLUTION to the equation

$-4x+1=13$

Example 2

Is $\frac{2}{5}=x$

a solution to the equation

$19=15x+8$

replace x with $\frac{2}{5}$ in

$19=15x+8$

this yeilds

$19=15\left(\frac{2}{5}\right)+8$

which reduces to

$19=14$

so 5 = x

IS NOT A SOLUTION to the equation

$19=15x+8$

Finding the Solution to an Equation
Solving for x

An equation is considered **solved** when the original equation has been changed into an equation with **x all alone** on one side of the = symbol and a **number on the other side**. The following four properties state the four operations that can be performed on any equation to produce a solution.

Solving 1 Step Equations

One Step Equations are equations that require the use of **one of the four** properties of equality.

The Addition Property of Equality

You can **add** the **same number** to **both sides** of an equation and still have an equivalent equation

If $x-3=10$

you add 3 to both sides

of $x-3=10$

$+3 \quad +3$

to get the solution

$x=13$

Check: $13-3=10$

The Subtraction Property of Equality

You can **subtract** the **same number** from **both sides** of an equation and still have an equivalent equation

If $4=x+9$

you subtract 9 from both sides

of $4=x+9$

$-9 \quad -9$

to get the solution

$-5=x$

Check: $4=-5+9$

The Multiplication Property of Equality

You can **multiply both sides** of an equation by the **same number** and still have an equivalent equation

If $5=\frac{x}{-3}$

you multiply both sides by -3

$(-3)5=\frac{x}{-3}(3)$

to get the solution

$-15=x$

Check: $5=\frac{-15}{-3}$

Note: Parenthesis ( ) must be used to show multiplication.

The Division Property of Equality

You can **divide both sides** of an equation by the **same number** and still have an equivalent equation

If $21=-7x$

you divide both sides by -7

$\frac{21}{-7}=\frac{-7x}{-7}$

to get the solution

$-3=x$

Check: $21=-7(-3)$

Note: A fraction bar must be used to show division.

Solving 2 Step Equations

Two Step Equations are equations that require the use of **two of the four** properties of equality to find a solution for the equation.

To solve for x (get x alone):

Step One: Eliminate the constant term: Add or Subtract

Step Two: Eliminate the coefficient. Multiply or divide.

Example 1

Solve for x

$3x - 4 = 8$

add 4 to
both sides of the equation

$3x - 4 = 8$
$+4 \quad +4$

$3x = 12$

divide both sides of the
equation by 3

$\frac{3x}{3} = \frac{12}{3}$

$\mathbf{x = 4}$

check: $3(4) - 4 = 8$

Example 2

Solve for x

$\frac{x}{2} - 5 = -1$

add 5 to
both sides of the equation

$\frac{x}{2} - 5 = -1$
$+5 \quad +5$

$\frac{x}{2} = 4$

multiply both sides of the equation by 2

$(2)\frac{x}{2} = 4(2)$

$\mathbf{x = 8}$

check: $\frac{8}{2} - 5 = -1$

$4 - 5 = -1$
$-1 = -1$

Solving Equations with an x term on both sides of the Equation

If the equation has a variable term appear **on both** sides of the equation, the solution takes **3 steps**.

Step 1: Eliminate the **smallest variable term** by adding or subtracting the smallest variable term to both sides of the equation. **You will now have a two step equation**

Step 2: **Eliminate the constant term** that is **on the side with the variable term** by adding or subtracting it to both sides of the equation.

Step 3: Eliminate the **coefficient with the variable** by multiplying or dividing both sides of the equation by the coefficient.

Example 1

$4x+6=2x+14$

Step 1 $-2x \quad -2x$

$2x+6=14$

Step 2 $-6 \quad -6$

$2x=8$

Step 3 $\frac{2x}{2}=\frac{8}{2}$

$\mathbf{x=4}$

Example 2

$3x+8=6x+14$

Step 1 $-3x \quad -3x$

$8=3x+14$

Step 2 $-14 \quad -14$

$-6=3x$

Step 3 $\frac{-6}{3}=\frac{3x}{3}$

$\mathbf{-2=x}$

Example 3

$x+6=-2x+15$

Step 1 $+2x \quad +2x$

$3x+6=15$

Step 2 $-6 \quad -6$

$3x=9$

Step 3 $\frac{3x}{3}=\frac{9}{3}$

$\mathbf{x=3}$

It is not required that you eliminate the smallest variable term first. You could choose to eliminate the largest variable term first.

Example 16A ⟵ **same problem** ⟶ **Example 16B**

eliminate the 3x on both sides

$5x+10=3x+16$

Step 1 $-3x \quad -3x$

$2x+10=16$

Step 2 $-10 \quad -10$

$2x=6$

Step 3 $\frac{2x}{2}=\frac{6}{2}$

$\mathbf{x=3}$

eliminate 5x on both sides

$5x+10=3x+16$

Step 1 $-5x \quad -5x$

$10=-2x \quad 16$

Step 2 $-16 \quad -16$

$-6=-2x$

Step 3 $\frac{-6}{-2}=\frac{-2x}{-2}$

$\mathbf{3=x}$

The solution is the same in both cases.

Does Every Equation Have Only One Solution?

It would seem from all the equations that we have solved that every equation has exactly one value for x as a solution. Actually, there are two cases when an equation does not have exactly one solution. In one case there are **no numbers** that can be substituted in for x to make the equation true. In a second case any number you chose can be substituted in for x and it will be a solution for the equation so **all numbers** are a solution.

Equations With No Numbers As A Solution

When you are getting the x terms on the same side of an equation and the **x terms cancel out** leaving you a statement with only numbers and that statement is **FALSE** then the answer is No Solution or $\varnothing$

Example 17

Solve For x: $4x+6=4x+14$

$4x+6=4x+14$ (subtract $4x$ from
$-4x \quad -4x$ both sides)

$6=14$

the x terms have dropped out
and the the remaining statement
is FALSE so there are no numbers
that work and the answer is
No Solution

Example 18

Solve For x: $-2x-5=-2x+8$

$-2x-5=-2x+8$ (add $2x$ to
$+2x \quad +2x$ both sides)

$-5=8$

the x terms have dropped out
and the the remaining statement
is FALSE so there are no numbers
that work and the answer is
No Solution

Equations With All Numbers As A Solution

When you are getting the x terms on the same side of an equation and the **x terms cancel out** leaving you a statement with only numbers in it and that statement is **TRUE** then the answer is

All Real Numbers (ARN) or Infinite Solutions

Example 19

Solve for x: $5x-8=5x-8$

$5x-8=5x-8$ (subtract $5x$ from
$-5x \quad -5x$ both sides)

$-8=-8$

the x terms have dropped out
and the the remaining statement
is TRUE so the answer is
All Numbers Work

Example 20

Solve for x: $-3x+4=-3x+4$

$-3x+4=-3x+4$ (add $3x$ to
$+3x \quad +3x$ both sides)

$4=4$

the x terms have dropped out
and the the remaining statement
is TRUE so the answer is
All Numbers Work

Distributive Equations

In some cases a **distributive step must be performed first** and then proceed to solve for x.

Example 1

$$-2(x-3)=-8$$

distribute the -2

$$-2x+6=-8$$

subtract 6 from both sides

$$-2x+6=-8$$
$$-6\quad -6$$

$$-2x=-14$$

divide both sides of the equation by -2

$$\frac{-2x}{-2}=\frac{-14}{-2}$$

$$\mathbf{x=7}$$

check: x = 7

$$-2(7-3)=-8$$
$$-2(4)=-8$$
$$-8=-8$$

Like Term Equations

In some cases **Like Terms on the same side of the equation must be combined first.**

Example 2

$$-15+6+2x=-3+3x+x$$

combine the -15 and 6 on the left
combine the 3x and x on the right

$$-9+2x=-3+4x$$

subtract 2x from both sides of the equation

$$-9+2x=-3+4x$$
$$-2x\qquad -2x$$

$$-9=-3+2x$$

add 3 to both sides of the equation

$$-9=-3+2x$$
$$+3\quad +3$$

$$-6=2x$$

divide bith sides of the equation by 2

$$\frac{-6}{2}=\frac{2x}{2}$$

$$\mathbf{-3=x}$$

Solving Equations with Fractions

Solving Equations with Fractions

Some or all of the terms in an equation may contain fractions. The multiplication property of equality allows us to multiply **every term in an equation by the same number.** If we chose the **Lowest Common Denominator** as the number we multiply each term by then the resulting equation will not have any fractions in it. We can then solve this equation by the methods of the previous two sections.

Solving Equations with Fractions

Step 1: Multiply each term of the equation by the Lowest Common Denominator (LCD). This will give you an equation without any fractions.

Step 2. Solve the equation.

1) Distribute if there is a distribute part of the equation.

2) Get the x terms all on one side of the equation by adding or subtracting.

3) **Eliminate the constant term** that is **on the side with the variable term** by adding or subtracting it to both sides of the equation.

4) Eliminate the **coefficient in front of the e variable** by multiplying or dividing both sides of the equation by the coefficient.

Example 1
The LCD is 6

$$\frac{2x}{3}-\frac{1}{2}=\frac{5}{6} \quad \left(\begin{array}{c}\text{multiply each}\\ \text{term by 6}\end{array}\right)$$

$$\left(\frac{6}{1}\right)\frac{2x}{3}-\left(\frac{6}{1}\right)\frac{1}{2}=\left(\frac{6}{1}\right)\frac{5}{6}$$

$$4x-3=5 \quad \text{(add 3 to both sides)}$$
$$+3 \quad +3$$

$$4x=8 \quad \text{(divide both sides by 4)}$$

$$\frac{4x}{4}=\frac{8}{4}$$
$$\mathbf{x=2}$$

Example 2
The LCD is 10

$$\frac{4x}{5}+\frac{3}{2}=\frac{3x}{10} \quad \left(\begin{array}{c}\text{multiply each}\\ \text{term by 10}\end{array}\right)$$

$$\left(\frac{10}{1}\right)\frac{4x}{5}+\left(\frac{10}{1}\right)\frac{3}{2}=\left(\frac{10}{1}\right)\frac{3x}{10}$$

$8x+15=3x$ (subtract 3x from
$-3x \quad -3x$ both sides)

$5x+15=0$ (subtract 15 from both sides)

$\frac{5x}{5}=\frac{-15}{5}$ (divide both sides by 5)
x = –3

Example 3

$$\frac{x-2}{3}=\frac{x}{2}-\frac{4}{3} \quad \left(\begin{array}{c}\text{multiply each}\\ \text{term by 6}\end{array}\right)$$

$$\left(\frac{6}{1}\right)\frac{(x-2)}{3}=\left(\frac{6}{1}\right)\frac{x}{2}-\left(\frac{6}{1}\right)\frac{4}{3}$$

$2(x-2)=3x-8$ (distribute)

$2x-4=3x-8$ (subtract 2x from
$-2x \quad -2x$ both sides)

$-4=x-8$ (add 8 to both sides)
$+8 \quad +8$
x = 4

Example 4
watch the – sign

$$\frac{2}{3}-\frac{x+1}{2}+=\frac{x}{4} \quad \left(\begin{array}{c}\text{multiply each}\\ \text{term by 12}\end{array}\right)$$

$$\left(\frac{12}{1}\right)\frac{2}{3}-\left(\frac{12}{1}\right)\frac{(x+1)}{2}=\left(\frac{12}{1}\right)\frac{x}{4}$$

$8-6(x+1)=3x$ (distribute)

$8-6x-6=3x$ (add like terms)

$-6x+2=3x$ (add -6x to
$+6x \quad +6x$ both sides)

$2=9x$

$\frac{2}{9}=\frac{9x}{9x}$ (divide both sides by 9)
$\frac{2}{9}=x=$

