Section 0 – 5: Evaluating Algebraic Expressions

Algebraic Expressions

An algebraic expression is any expression that combines numbers and letters with a set of operations on those numbers and letters. **An expression does not contain an = sign.**

Vocabulary for Algebraic Expressions

Term: Each part of an expression that is separated by a + or – sign is one **term** in the expression.

In the expression $2x+5$ T he $2x$ and the $+5$ are each separate terms

In the expression $3x^2-2x-7$ the $3x^2$ and the $-2x$ and the –7 are each separate terms

Constant Term: A number by itself with no variable is a **constant term.** The term is called **constant** because its value is known and stays the same.

$2x+5$ the 5 is a constant term

$3x^2-2x-7$ the –7 is a constant term

Variable Term: Each letter is called a **variable**. Each term with a variable is a variable term

in $2x+5$ the 2x is a variable term

In $3x^2-2x-7$ the $3x^2$ and the $-2x$ are variable terms

Coefficient of a Term: A number in front of a variable is called a **coefficient**.

in $2x+5$ the 2 is a coefficient of x

In $3x^2-7$ the 3 is a coefficient of x^2

Exponent: The number above the variable is called an **exponent** (or power) of the variable.

$3x^2$ the variable x has an exponent of 2

$2x^3$ the variable x has an exponent of 3

$5x$ the variable x has with an exponent of 1. If the variable has an exponent of 1 it is not shown.

Example: $-4x^2+2y-8$

The expression $-4x^2+2y-8$ has 3 terms. $-4x^2$ and $+2y$ and -8 are each separate terms.

The $-4x^2$ is a variable term with a coefficient of –4 and the variable x has an exponent of 2

The $+2y$ is a variable term with a coefficient of +2 and the variable y has an exponent of 1

The -8 is a constant term.

Reading an algebraic expression

This chapter will involve expressions or terms that have a constant in front of a variable and an exponent above the variable. Each term with a constant and variables written next to each other are understood to be a product by the way the term is written and read.

Example 1

$5x$ is read
five x
and means
5 times x

Example 2

$-2x$ is read
negitive two x
and means
-2 times x

Example 3

$-x$ is read
negitive x
and means
-1 times x

Example 4

$-4x^3$ is read
negitive four x to the third
and means
4 times x times x times x

Example 5

$5x^2y$ is read
5 x squared y
and means
5 times x times x times y

Example 6

$-xy^2$ is read
negitive x y squared
and means
-1 times x times y times y

Evaluation of Algebraic Expressions

A number by itself with no variable is a **constant term.** A constant term is called **constant** because its value is known and stays the same. In the expression $2x+5$ the 5 is a constant term. The value of the constant is fixed as 5 and cannot change. Terms with letters or letters and numbers are called **variable terms.** Variable terms are called variable because they have a letter or letters whose value may change or vary.

If the value for each variable is known then the value for the entire expression can be found. A given value for each variable can be **substituted** in for that letter to create expressions like the ones in Chapter 3. The value of the expression is then determined by using the rules for the order of operations PEMDAS.

The process of **substituting** values into the expression for each variable and then determining the number value for the expression is called **Evaluating the Expression.**

The process of **substituting** values into the expression for each variable involves creating a set of parenthesis for each variable letter and then **substituting** the values for each letter into the parenthesis. The final process of **Evaluating the Expression** involves simplifying the resulting expression using the rules for the order of operations PEMDAS. The **Steps to Evaluate Algebraic Expressions** are given next.

Steps to Evaluate an Algebraic Expression

Step 1. Rewrite the expression with each letter inside a separate parenthesis.

Step 2. Replace each variable with the value given for it (substitution).

Step 3. Use the order of operation rules (PEMDAS) to find the value of the expression.

If $A = -3$ and $B = 2$ and $C = 4$ and $D = -2$ then evaluate the expression.

Example 1

$$2A + 3B$$

means $2(A) + 3(B)$

$$2(-3) + 3(4)$$

$$-6 + 12$$

$$= \mathbf{6}$$

Example 2

$$4A - C$$

means $(A) - (C)$

$$4(-3) - (-2)$$

$$-12 + 2$$

$$= \mathbf{-10}$$

Example 3

$$5B - 2A^2$$

means $5(B) - 2(A)^2$

$$5(2) - 2(-3)^2$$

$$10 - 18$$

$$= -8$$

Example 4

$$\frac{-3C + A}{A + D} \text{ means } \frac{-3(4) + (-3)}{-3 + (-2)}$$

$$= \frac{-12 - 3}{-3 - 2}$$

$$= \frac{-15}{-5} = -3$$

Example 5

$$\frac{5C - 3B}{B - C} \text{ means } \frac{5(4) - 3(2)}{2 - (4)}$$

$$= \frac{20 - 6}{2 - 4}$$

$$= \frac{14}{-2} = -7$$

Evaluation of Expressions with x and y

Many expressions in algebra use x and y as the variables instead of the A,B or C used so far in this chapter. It is also common to change the values for the variables with each problem rather than keeping the values the same for several problems. The problems below are completed the same way as the earlier problems with A, B and C but use x and y variables instead and change the value of x and y in each problem.

Example 6

Evaluate $-x^2 - 3x - 5$ for $x = -4$

$$-(-4)^2 - 3(-4) - 5$$

$$= -(16) - 3(-4) - 5$$

$$= -16 + 12 - 5$$

$$= -9$$

Example 7

Evaluate $-2xy + 2y^2$ for $x = -2$ and $y = 3$

$$-2(-2)(3) + 2(3)^2$$

$$= -2(-2)(3) + 2(9)$$

$$= 12 + 18$$

$$= 30$$

Example 8

Evaluate $-8x+6y$ for $x=\frac{3}{4}$ and $y=\frac{5}{2}$

$$\frac{-8}{1}\left(\frac{3}{4}\right)+\frac{6}{1}\left(\frac{5}{2}\right)$$

$$=\frac{-24}{4}+\frac{30}{2}$$

$$=-6+15$$

$$=9$$

Example 9

Evaluate $\frac{x-y}{2x+y}$ for $x=3$ and $y=-4$

$$\frac{3-(-4)}{2(3)+(-4)}$$

$$=\frac{3+7}{6-4}$$

$$=\frac{10}{2}$$

$$=5$$

Evaluation of Formulas Introduction

There are many problems in mathematics that use formulas with variables to express the relationship between the different variables in the formula. The value for the unknown can be found by substituting the given given values in for each variable and then evaluating the expression.

Convert Fahrenheit into Celsius

Find the temperature in C if

$$\text{C}=\frac{5(\text{F}-32)}{9}$$

and F = 77 degrees F

$$\text{C}=\frac{5(77-32)}{9}$$

$$\text{C}=\frac{5(45)}{9}=25^{\circ}\text{C}$$

Convert Celsius into Fahrenheit

Find the temperature in F if

$$\text{F}=\frac{9\text{C}}{5}+32$$

and C = 75 degrees C

$$\text{F}=\frac{9\cdot 75}{5}+32$$

$$\text{F}=135+32=167^{\circ}\text{F}$$

The Area of a Triangle

Find the area of a triangle if

$$\text{A}=\frac{1}{2}\text{bh}$$

and b = 12 ft., h = 5 ft.

$$\text{A}=\frac{1}{2}(12)(5)$$

A = 30 sq. ft.

The Area of a Trapezoid

Find the area of a trapezoid if

$$\text{A}=\frac{\text{h}(B+b)}{2}$$

and h = 6 ft., B = 2 ft. b = 7 ft.

$$\text{A}=\frac{6(2+7)}{2}$$

$$\text{A}=\frac{6(9)}{2}=\frac{54}{2}$$

A = 27 sq. ft.

The Volume of a Cube

Find the volume of a Cube if

$V = s^3$

and $s = \frac{1}{2}$ inches

$V = \left(\frac{1}{2}\right)^3$

$V = \frac{1}{8}$ cubic inches

The Volume of a Square Pyramid

Find the volume of a Square Pyramid

if $V = \frac{1}{3} \cdot s^2 \cdot h$

and s = 3 inches and h = 2 inches

$V = \frac{1}{3} \cdot 3^2 \cdot 2$

$V = \frac{1}{3} \cdot 9 \cdot 2$

$V = 6$ cubic inches

Geometric Formulas with π Introduction

The symbol π stands for an irrational number. An irrational number is a decimal number that **never ends or repeats**. If you want an answer that is exact then the answer must must be written with the symbol π in it.

The Volume of a Cone

Find the exact volume of a cone

if $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$

and r = 4 ft. and h = 3 ft.

$V = \frac{1}{3} \cdot \pi \cdot 4^2 \cdot 3$

$V = \frac{1}{3} \cdot \pi \cdot 48$

$V = 16\pi$ cubic feet

The Volume of a Sphere

Find the exact volume of a sphere

if $V = \frac{4}{3} \cdot \pi \cdot r^3$

and r = 2 feet

$V = \frac{4}{3} \cdot \pi \cdot 2^3$

$V = \frac{4}{3} \cdot \pi \cdot 8$

$V = \frac{32\pi}{3}$ cubic feet

Volume of a Right Circular Cylinder

Find the exact volume of a circular cylinder

if $V = \pi \cdot r^2 \cdot h$

and r = 2 in. and h = 3 in.

$V = \pi \cdot 2^2 \cdot 3$

$V = \pi \cdot 4 \cdot 3$

$V = 12\pi$ cubic inches

The Surface Area of a Cone

Find the exact Surface Area of a cone

if $SA = \pi \cdot r \cdot s + \pi \cdot r^2$

and r = 2 ft. ands = 3 ft.

$SA = \pi \cdot 2 \cdot 3 + \pi \cdot 3^2$

$V = 6\pi + 9\pi$

$V = 15\pi$ square feet

