

Section 0 – 4: The Order of Operations

There is an agreed upon order in which the operations of exponents, multiplication, division, addition and subtraction are performed if there are several different operations in one problem.

We use the mnemonic **P.E.M.D.A.S** to help us remember the order of operations.

Please **E**xcuse **M**y **D**ear **A**unt **S**ally

1. **P** Simplify **inside the parentheses and get one number**.
2. **E** Simplify any **base with an exponent**.
3. **M,D** Simplify any multiply and divide problems.
4. **A,S** Simplify the addition and/or subtraction of signed numbers.

Parentheses

The first thing we do is to perform any operations that are **inside the parentheses**. We **keep the answer** to this step **inside the parentheses**. If there is just one number inside the parentheses we do not perform this step.

$$(5 - 7) \text{ is replaced by } (-2) \quad (9 - 2) \text{ is replaced by } (7) \quad (-3 - 6) \text{ is replaced by } (-9)$$

Exponents

The next operation performed is with any number that has an **exponent** above it. You multiply a **positive base** times itself the number of times as the exponent.

$$5^2 = (5)(5) = 25 \quad 2^3 = (2)(2)(2) = 8 \quad 2^4 = (2)(2)(2)(2) = 16$$

You multiply a **negative base inside a parentheses** times itself the number of times as the exponent. The negative sign in front of the base **is included** with the base if the negative sign is inside the parentheses.

$$(-3)^2 = (-3)(-3) = 9 \quad (-3)^2 = (-3)(-3) = 9 \quad (-2)^3 = (-2)(-2)(-2) = -8$$

Important Note: A negative sign **in front** of the base is **not included with the base** if the base is not inside a parentheses. The negative sign in front of the base is understood to mean **multiply the base without the negative sign times itself the number of times as the exponent first and then multiply that number by -1**.

$$\begin{array}{lll} -2^2 = -1(2)^2 = -4 & -3^2 = -1(3)^2 = -9 & -4^2 = -1(4)^2 = -16 \\ \text{the } - \text{ sign is NOT} & \text{the } - \text{ sign is NOT} & \text{the } - \text{ sign is NOT} \\ \text{included with the 2} & \text{included with the 3} & \text{included with the 4} \end{array}$$

Multiply and Divide

Multiply a number in front of a parenthesis by the number inside the parentheses. Perform any division problems written as written as $8 \div 4$.

Add and Subtract

After all the multiplication and division is complete there will only be numbers with a + or – sign between them. Add or subtract the signed numbers to complete the process.

Order Of Operations Examples

Example 1

$$7 - 2(5 - 2) \quad \mathbf{P}$$

$$7 - 2(3) \quad \mathbf{M}$$

$$7 - 6 \quad \mathbf{S}$$

$$\mathbf{1}$$

Example 2

$$4 - 5(2 - 6) \quad \mathbf{P}$$

$$4 - 5(-4) \quad \mathbf{M}$$

$$4 + 20 \quad \mathbf{A}$$

$$\mathbf{24}$$

Example 3

$$4(2) + 5(2 - 3) \quad \mathbf{P}$$

$$4(2) + 10 \div 2 \quad \mathbf{M - D}$$

$$8 - 5 \quad \mathbf{S}$$

$$\mathbf{3}$$

Example 4

$$(-3)^2 - 4^2 \quad \mathbf{E}$$

$$9 - 16 \quad \mathbf{S}$$

$$\mathbf{-7}$$

Example 5

$$-4^2 - 3^2 \quad \mathbf{E}$$

$$-16 - 9 \quad \mathbf{A}$$

$$\mathbf{-25}$$

Example 6

$$-2(3)^2 - 2^3 \quad \mathbf{E}$$

$$-2(9) - 8 \quad \mathbf{M}$$

$$-18 - 8 \quad \mathbf{A}$$

$$\mathbf{-26}$$

Example 7

$$(3 + 1)^2 - 2(5 - 2)^2 \quad \mathbf{P}$$

$$(4)^2 - 2(3)^2 \quad \mathbf{E}$$

$$(16) - 2(9) \quad \mathbf{M}$$

$$16 - 18 \quad \mathbf{S}$$

$$\mathbf{-2}$$

Example 8

$$2(2 - 5)^2 + (7 - 3)^2 \quad \mathbf{P}$$

$$2(-3)^2 + (4)^2 \quad \mathbf{E}$$

$$2(9) + (16) \quad \mathbf{M}$$

$$18 + 16 \quad \mathbf{A}$$

$$\mathbf{34}$$

Example 9

$$-(4 + 1)^2 - 3^2 \quad \mathbf{P}$$

$$-(5)^2 - 9 \quad \mathbf{E}$$

$$-(25) - 9 \quad \mathbf{M}$$

$$-25 - 9 \quad \mathbf{A}$$

$$\mathbf{-34}$$

The Order of Operations with Fractions

The Numerator (top) and Denominator (bottom) of a fraction **are considered separate expressions that are inside their own set of parentheses**. The Order of Operations PEMDAS requires that we simplify the Numerator and Denominator (separately and then reduce the final fraction if possible).

$$\frac{\text{(the numerator must be simplified separately)}}{\text{(the denominator must be simplified separately)}}$$

1. Simplify the Numerator and Denominator separately using **PEMDAS**.
2. Reduce the final fraction.

Example 19

$$\frac{2(3+1)}{5(4-2)} \quad \mathbf{P}$$

$$\frac{2(4)}{5(2)} \quad \mathbf{M}$$

$$= \frac{8}{10}$$

$$= \frac{4}{5}$$

Example 20

$$\frac{-4(2-8)}{-4+2} \quad \mathbf{P}$$

$$\frac{-4(-6)}{-4+2} \quad \mathbf{M}$$

$$\frac{24}{-2}$$

$$= -12$$

Example 21

$$\frac{-4(2)-2}{2(-4+2)} \quad \mathbf{P}$$

$$= \frac{-4(2)-2}{2(-2)} \quad \mathbf{M}$$

$$= \frac{-8-2}{-4} \quad \mathbf{A}$$

$$= \frac{-10}{-4}$$

$$= \frac{5}{2}$$