

Section 9 – 7: Solving Radical Equations

Radical Equations are equations that have a radical expression in one or more of the terms in the equation. We must find a way to eliminate the square root part of the equation. We know that a pair of the same factors under a square root reduces to a single factor without a square root.

Squaring a square root expression gives an expression without a radical sign

$$(\sqrt{3x})^2 = \sqrt{3x} \cdot \sqrt{3x} = 3x$$

$$(\sqrt{2x-3})^2 = \sqrt{2x-3} \cdot \sqrt{2x-3} = 2x-3$$

How to Solve Radical Equations with Square Roots

1. Get the term with the square root **alone one one side of the equation.**
2. If the isolated square root is equal to a negative number then there are no Real Numbers that will work. We write NRN as the answer
3. Square both sides.
4. Solve for x
5. Check your answer(s). If no answers work there is No Real Number that is a solution.

Example 1

Solve for x

$$\sqrt{2x} + 8 = 14$$

Get the square root alone on one side of the equation by subtracting 8 from both sides

$$\sqrt{2x} = 6$$

$$(\sqrt{2x})^2 = 6^2$$

$$2x = 36 \text{ solve for } x$$

$$x = 18$$

Check:

$$\sqrt{2(18)} + 8 = 14$$

$$\sqrt{36} + 8 = 14$$

$$6 + 8 = 14$$

Example 2

Solve for x

$$\sqrt{8x} - 5 = -1$$

Get the square root alone on one side of the equation by adding 5 to both sides

$$\sqrt{8x} = 4$$

$$(\sqrt{8x})^2 = 4^2$$

$$8x = 16 \text{ solve for } x$$

$$x = 2$$

Check:

$$\sqrt{8(2)} - 5 = -1$$

$$\sqrt{16} - 5 = -1$$

$$4 - 5 = -1$$

Example 3

Solve for x

$$\sqrt{2x} + 7 = 3$$

Get the square root alone on one side of the equation by subtracting 7 from both sides

$$\sqrt{2x} = -4$$

STOP: An isolated square root cannot equal a negative number so the answer is NRN

Had you continued you would have found that

$$x = 8$$

but that answer would not have been true if you checked it by putting it back into the equation.

Example 4

Solve for x

$$\sqrt{3x-5} = 5$$

square both sides

$$(\sqrt{3x-5})^2 = (5)^2$$

$$3x - 5 = 25 \text{ solve for } x$$

$$3x = 30$$

$$x = 10$$

Check:

$$\sqrt{3(10)-5} = 5$$

$$\sqrt{30-5} = 5$$

$$\sqrt{25} = 5$$

Example 7

Solve for x

$$\sqrt{3x-5} = \sqrt{5x-9}$$

square both sides

$$(\sqrt{3x-5})^2 = (\sqrt{5x-9})^2$$

$$3x - 5 = 5x - 9 \text{ solve for } x$$

$$4 = 2x$$

$$2 = x$$

Check:

$$\sqrt{3(2)-5} = \sqrt{5(2)-9}$$

$$\sqrt{6-5} = \sqrt{10-9}$$

$$\sqrt{1} = \sqrt{1}$$

Example 5

Solve for x

$$\sqrt{4x+13} = 3$$

square both sides

$$(\sqrt{4x+13})^2 = (3)^2$$

$$4x + 13 = 9 \text{ solve for } x$$

$$4x = -4$$

$$x = -1$$

Check:

$$\sqrt{4(-1)+13} = 3$$

$$\sqrt{-4+13} = 3$$

$$\sqrt{9} = 3$$

Example 8

Solve for x

$$\sqrt{6x-1} = \sqrt{2x+11}$$

square both sides

$$(\sqrt{6x-1})^2 = (\sqrt{2x+11})^2$$

$$6x - 1 = 2x + 11 \text{ solve for } x$$

$$4x = 12$$

$$x = 3$$

Check:

$$\sqrt{6(3)-1} = \sqrt{2(3)+11}$$

$$\sqrt{18-1} = \sqrt{6+11}$$

$$\sqrt{17} = \sqrt{17}$$

Example 6

Solve for x

$$\sqrt{3x+1} = 3$$

square both sides

$$(\sqrt{3x+1})^2 = (3)^2$$

$$3x + 1 = 9 \text{ solve for } x$$

$$3x = 8$$

$$x = \frac{8}{3}$$

Check:

$$\sqrt{3\left(\frac{8}{3}\right)+1} = 3$$

$$\sqrt{8+1} = 3$$

Example 9

Solve for x

$$\sqrt{5x+2} = \sqrt{2x-7}$$

square both sides

$$(\sqrt{5x+2})^2 = (\sqrt{2x-7})^2$$

$$5x + 2 = 2x - 7 \text{ solve for } x$$

$$3x = -9$$

$$x = -3$$

Check:

$$\sqrt{5(-3)+2} = \sqrt{2(-3)-7}$$

$$\sqrt{-15+2} = \sqrt{-6-7}$$

$$\sqrt{-13} = \sqrt{-13}$$

We cannot have the Square Root of a negative so NRN