

## Section 9 – 5B:

### Rationalizing the Denominator of a Fraction with a Binomial Expression where one or both terms contains a Square Root

Each of following fractions has a binomial expression in the denominator. Each denominator has a square root in one or both of its terms .

#### Example 1

$$\frac{3}{4 - \sqrt{3}}$$

#### Example 2

$$\frac{5}{\sqrt{7} - 2}$$

#### Example 3

$$\frac{3}{\sqrt{7} - \sqrt{3}}$$

It is common to require that the denominator not contain any radicals. The process of eliminating radicals from the denominator of a fraction is called **rationalizing the denominator**. In the previous section we had a monomial term with a square root in the denominator. In that case we multiplied the top and bottom of the fraction by the square root. This technique will not work with a binomial.

Multiplying the fraction by  $\frac{\sqrt{3}}{\sqrt{3}}$  does not rationalize the denominator

$$\frac{5}{4 - \sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{4\sqrt{3} - \sqrt{9}} = \frac{5\sqrt{3}}{4\sqrt{3} - 3}$$

multiplying the bottom by  $\sqrt{3}$  simply changes which term in the denominator has the square root.

### The Square Root Conjugate

When we FOIL some binomials we end up with a whole number. This happens if the binomials are the **sum and difference of the same terms**. The pair is called a **Square Root Conjugate**

#### Example 4

$$(5 + \sqrt{3})(5 - \sqrt{3})$$

$$= 25 - 2\sqrt{3} + 2\sqrt{3} - \sqrt{9}$$

the middle terms cancel

$$= 25 - \sqrt{9} = 25 - 3$$

$$= 22$$

#### Example 5

$$(6 - \sqrt{5})(6 + \sqrt{5})$$

$$= 36 - 7\sqrt{5} + 7\sqrt{5} - \sqrt{25}$$

the middle terms cancel

$$= 36 - \sqrt{25} = 36 - 25$$

$$= 11$$

#### Example 6

$$(\sqrt{7} - \sqrt{2})(\sqrt{7} + \sqrt{2})$$

$$= \sqrt{49} - \sqrt{14} + \sqrt{14} - \sqrt{4}$$

the middle terms cancel

$$= 7 - 2$$

$$= 5$$

### Rationalizing the Denominator

Rationalize the Denominator by multiplying by the binomial that will form a Conjugate with the denominator and then simplify the fraction as shown in the example on the following pages.

$$\frac{2}{7 + \sqrt{3}} \text{ multiply by } \frac{7 - \sqrt{3}}{7 - \sqrt{3}}$$

$$\frac{9}{5 - \sqrt{2}} \text{ multiply by } \frac{5 + \sqrt{2}}{5 + \sqrt{2}}$$

**Example 7**

Simplify  $\frac{5}{4-\sqrt{3}}$

multiply the top and bottom by  $4+\sqrt{3}$ 

$$= \frac{5}{4-\sqrt{3}} \cdot \frac{4+\sqrt{3}}{4+\sqrt{3}}$$

Note:  $(4-\sqrt{3})(4+\sqrt{3}) = 16-9 = 7$

$$= \frac{5(4+\sqrt{3})}{7} = \frac{20+5\sqrt{3}}{7}$$

**Example 8**

Simplify  $\frac{7}{5+\sqrt{2}}$

multiply the top and bottom by  $5-\sqrt{2}$ 

$$= \frac{7}{5+\sqrt{2}} \cdot \frac{5-\sqrt{2}}{5-\sqrt{2}}$$

Note:  $(5-\sqrt{2})(5+\sqrt{2}) = 25-2 = 23$

$$= \frac{7(5-\sqrt{2})}{23} = \frac{35-7\sqrt{2}}{23}$$

**Example 9**

Simplify  $\frac{8}{6+\sqrt{2}}$

multiply the top and bottom by  $6-\sqrt{2}$ 

$$= \frac{8}{6+\sqrt{2}} \cdot \frac{6-\sqrt{2}}{6-\sqrt{2}}$$

Note:  $(6+\sqrt{2})(6-\sqrt{2}) = 36-2 = 34$

$$= \frac{8(6-\sqrt{2})}{34}$$

the 8 outside the brackets  
and the 34 are both factors  
and can reduce each other

$$= \frac{8^4(6-\sqrt{2})}{34^{17}}$$

$$= \frac{4(6-\sqrt{2})}{17} = \frac{24-4\sqrt{2}}{17}$$

**Example 10**

Simplify  $\frac{12}{5-\sqrt{3}}$

multiply the top and bottom by  $5+\sqrt{3}$ 

$$= \frac{12}{5-\sqrt{3}} \cdot \frac{5+\sqrt{3}}{5+\sqrt{3}}$$

Note:  $(5-\sqrt{3})(5+\sqrt{3}) = 25-3 = 22$

$$= \frac{12(5+\sqrt{3})}{22}$$

the 12 outside the brackets  
and the 22 are both factors  
and can reduce each other

$$= \frac{12^6(5+\sqrt{3})}{22^{11}}$$

$$= \frac{6(5+\sqrt{3})}{11} = \frac{30+6\sqrt{3}}{11}$$

### Example 11

Simplify  $\frac{6}{\sqrt{7}-\sqrt{3}}$

multiply the top and bottom by  $\sqrt{7} + \sqrt{3}$

$$= \frac{6}{\sqrt{7}-\sqrt{3}} \cdot \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$$

Note:  $(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3}) = 7-3 = 4$

$$= \frac{6(\sqrt{7}+\sqrt{3})}{4}$$

the 6 outside the brackets  
and the 4 are both factors  
and can reduce each other

$$= \frac{6^3(\sqrt{7}+\sqrt{3})}{4^2}$$

$$= \frac{3(\sqrt{7}+\sqrt{3})}{2} = \frac{3\sqrt{7}+3\sqrt{3}}{2}$$

### Example 12

Simplify  $\frac{12}{\sqrt{11}+\sqrt{2}}$

multiply the top and bottom by  $\sqrt{11}-\sqrt{2}$

$$= \frac{12}{\sqrt{11}+\sqrt{2}} \cdot \frac{\sqrt{11}-\sqrt{2}}{\sqrt{11}-\sqrt{2}}$$

Note:  $(\sqrt{11}+\sqrt{2})(\sqrt{11}-\sqrt{2}) = 11-2 = 9$

$$= \frac{12(\sqrt{11}-\sqrt{2})}{9}$$

the 12 outside the brackets  
and the 9 are both factors  
and can reduce each other

$$= \frac{12^4(\sqrt{11}+\sqrt{2})}{9^3}$$

$$= \frac{4(\sqrt{11}-\sqrt{2})}{3} = \frac{4\sqrt{11}-4\sqrt{2}}{3}$$

**Example 13**

Simplify  $\frac{\sqrt{3}}{4 + \sqrt{2}}$

multiply the top and bottom by  $4 - \sqrt{2}$

$$= \frac{\sqrt{3}}{4 + \sqrt{2}} \cdot \frac{4 - \sqrt{2}}{4 - \sqrt{2}}$$

Note:  $(4 - \sqrt{2})(4 - \sqrt{2}) = 16 - 2 = 14$

$$= \frac{\sqrt{3}(4 - \sqrt{2})}{14}$$

$$= \frac{4\sqrt{3} - \sqrt{6}}{14}$$

**Example 15**

Simplify  $\frac{\sqrt{2}}{3 - \sqrt{6}}$

multiply the top and bottom by  $3 + \sqrt{6}$

$$= \frac{\sqrt{2}}{3 - \sqrt{6}} \cdot \frac{3 + \sqrt{6}}{3 + \sqrt{6}}$$

Note:  $(3 + \sqrt{6})(3 + \sqrt{6}) = 9 - 6 = 3$

$$= \frac{\sqrt{2}(3 + \sqrt{6})}{3}$$

$$= \frac{2\sqrt{3} + \sqrt{12}}{3} = \frac{2\sqrt{3} + \sqrt{4 \cdot 3}}{3}$$

$$= \frac{2\sqrt{3} + 2\sqrt{3}}{3} = \frac{2\sqrt{3} + 2\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$$

**Example 14**

Simplify  $\frac{\sqrt{7}}{\sqrt{5} - \sqrt{2}}$

multiply the top and bottom by  $\sqrt{5} + \sqrt{2}$

$$= \frac{\sqrt{7}}{\sqrt{5} - \sqrt{2}} \cdot \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} + \sqrt{2}}$$

Note:  $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = 5 - 2 = 3$

$$= \frac{\sqrt{7}(\sqrt{5} + \sqrt{2})}{3}$$

$$= \frac{\sqrt{35} + \sqrt{14}}{3}$$

**Example 16**

Simplify  $\frac{\sqrt{3}}{\sqrt{15} + \sqrt{6}}$

multiply the top and bottom by  $\sqrt{15} - \sqrt{6}$

$$= \frac{\sqrt{3}}{\sqrt{15} + \sqrt{6}} \cdot \frac{\sqrt{15} - \sqrt{6}}{\sqrt{15} - \sqrt{6}}$$

Note:  $(\sqrt{15} - \sqrt{6})(\sqrt{15} - \sqrt{6}) = 15 - 6 = 9$

$$= \frac{\sqrt{3}(\sqrt{15} - \sqrt{6})}{9}$$

$$= \frac{\sqrt{45} + \sqrt{18}}{9} = \frac{\sqrt{9 \cdot 5} + \sqrt{9 \cdot 2}}{9}$$

$$= \frac{3\sqrt{5} + 3\sqrt{2}}{9} = \frac{\sqrt{5} + \sqrt{2}}{3}$$