Section 9–5A: Rationalizing the Denominator of a Fraction with a Monomial Term that Contains a Square Root

Each of following factions has a **monomial** expression in the **denominator** that contains a **square root** or a **square root** multiplied by a number.

$\frac{\sqrt{54}}{\sqrt{24}}$ $\frac{6\sqrt{12}}{8\sqrt{3}}$ $\frac{8}{\sqrt{6}}$ $\frac{10\sqrt{7}}{\sqrt{5}}$

It is common to require that the denominator of a fraction not contain any radicals. The process of eliminating radicals from the denominator of a fraction is called **rationalizing the denominator**.

Reducing Factions with Monomial terms like $\frac{\sqrt{A}}{\sqrt{B}}$

Many fractions with square roots in both the numerator and denominator can be simplified by reducing the numbers under the square roots. Fractions with only monomial terms that are each under a square root like $\frac{\sqrt{A}}{\sqrt{B}}$ may be **reduced** if A and B have a common factor. If the numbers under the square roots have a common factor then the numbers under the square roots can **reduce each other**.

Example 1

Simplify $\frac{\sqrt{12}}{\sqrt{3}}$

$\frac{\sqrt{12}}{\sqrt{3}}$

the 12 and 3 are both factors under a radical sign so they can reduce each other

$= \frac{\sqrt{\cancel{12}^{\,4}}}{\sqrt{\cancel{3}^{\,1}}} = \frac{\sqrt{4}}{\sqrt{1}}$

$= \frac{2}{1} = 2$

Example 2

Simplify $\frac{\sqrt{50}}{\sqrt{8}}$

$\frac{\sqrt{50}}{\sqrt{8}}$

the 50 and 8 are both factors under a radical sign so they can reduce each other

$\frac{\sqrt{\cancel{50}^{\,25}}}{\sqrt{\cancel{8}^{\,4}}} = \frac{\sqrt{25}}{\sqrt{4}}$

$= \frac{5}{2}$

Reducing Factions with Monomial terms like $\frac{C\sqrt{A}}{D\sqrt{B}}$

Fractions with only monomial terms like $\frac{C\sqrt{A}}{D\sqrt{B}}$ may be be **reduced**. If **A and B** have a common factor then the numbers under the square root can **reduce each othe**r. If **C and D** have a common factor then the numbers outside the square root can **reduce each other**.

Warning: A number under a square root and a number outside a square root **CAN NOT** reduce each other

Example 3

Simplify $\frac{10\sqrt{12}}{4\sqrt{27}}$

$\frac{10\sqrt{12}}{4\sqrt{27}}$

the 12 and 27 are both factors under a radical sign so they can reduce each other

the 10 and 4 are both factors outside a radical sign so they can reduce each other

$$\frac{\cancel{10}^{5}\sqrt{\cancel{12}^{4}}}{\cancel{4}^{2}\sqrt{\cancel{27}^{9}}} = \frac{5\sqrt{4}}{2\sqrt{9}}$$

$$= \frac{5 \bullet 2}{2 \bullet 3} = \frac{5 \bullet \cancel{2}}{\cancel{2} \bullet 3} = \frac{5}{3}$$

Example 4

Simplify $\frac{6\sqrt{27}}{9\sqrt{75}}$

$\frac{6\sqrt{27}}{9\sqrt{75}}$

the 27 and 75 are both factors under a radical sign so they can reduce each other

the 6 and 9 are both factors outside a radical sign so they can reduce each other

$$\frac{\cancel{6}^{2}\sqrt{\cancel{27}^{9}}}{\cancel{9}^{3}\sqrt{\cancel{75}^{25}}} = \frac{2\sqrt{9}}{3\sqrt{25}}$$

$$= \frac{2 \bullet 3}{3 \bullet 5} = \frac{2 \bullet \cancel{3}}{\cancel{3} \bullet 5} = \frac{2}{5}$$

Rationalizing the Denominator of a Fraction with a Monomial Term that contains a Square Root

Many fractions with square roots can not be simplified by reducing the numbers under the square roots as in the examples above.

$\frac{2}{\sqrt{3}}$ $\frac{7}{\sqrt{11}}$ $\frac{\sqrt{7}}{\sqrt{3}}$ $\frac{2\sqrt{5}}{\sqrt{6}}$

It is common to require that the denominator not contain any radicals. In fractions where the numbers under the square square roots cannot be reduced to eliminate the square root in the denominator we must find another process that will eliminate the radical from the denominator. The process of eliminating the radical from the denominator of a fraction is called **rationalizing the denominator**.

Multiplying the fraction $\frac{\sqrt{A}}{\sqrt{B}}$ **by** $\frac{\sqrt{B}}{\sqrt{B}}$ will eliminate the radical from the denominator

$\frac{7}{\sqrt{3}}$ multiply the top and bottom by $\sqrt{3}$

$\frac{5}{\sqrt{6}}$ multiply the top and bottom by $\sqrt{6}$

Example 5

Simplify $\frac{7}{\sqrt{3}}$

$\frac{7}{\sqrt{3}}$ multiply the top and bottom by $\sqrt{3}$

$= \frac{7}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}}$ the $\sqrt{3}$ times itself is 3

$= \frac{7\sqrt{3}}{3}$

Note: The 3 under the radical sign and the 3 outside the radical cannot reduce each other

Example 6

Simplify $\frac{5}{\sqrt{6}}$

$\frac{5}{\sqrt{6}}$ multiply the top and bottom by $\sqrt{6}$

$= \frac{5}{\sqrt{6}} \bullet \frac{\sqrt{6}}{\sqrt{6}}$ the $\sqrt{6}$ times itself is 6

$= \frac{5\sqrt{6}}{6}$

Note: The 6 under the radical sign and the 6 outside the radical cannot reduce each other

Example 7

Simplify $\frac{6}{\sqrt{10}}$

$\frac{6}{\sqrt{10}}$ multiply the top and bottom by $\sqrt{10}$

$= \frac{6}{\sqrt{10}} \bullet \frac{\sqrt{10}}{\sqrt{10}}$ the $\sqrt{10}$ times itself is 10

$= \frac{6\sqrt{10}}{10}$

the 6 and 10 are both factors outside a radical sign so they can reduce each other

$= \frac{\cancel{6}^{3}\sqrt{10}}{\cancel{10}^{5}} = \frac{3\sqrt{10}}{5}$

Example 8

Simplify $\frac{25}{\sqrt{15}}$

$\frac{25}{\sqrt{15}}$ multiply the top and bottom by $\sqrt{15}$

$= \frac{25}{\sqrt{15}} \bullet \frac{\sqrt{15}}{\sqrt{15}}$ the $\sqrt{15}$ times itself is 15

$= \frac{25\sqrt{10}}{15}$

the 25 and 15 are both factors outside a radical sign so they can reduce each other

$= \frac{\cancel{25}^{5}\sqrt{10}}{\cancel{15}^{3}} = \frac{5\sqrt{10}}{3}$

Example 9

Simplify $\frac{3\sqrt{2}}{\sqrt{5}}$

$\frac{3\sqrt{2}}{\sqrt{5}}$ multiply the top and bottom by $\sqrt{5}$

$= \frac{3\sqrt{2}}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}}$

$3\sqrt{2} \bullet \sqrt{5} = 3\sqrt{10}$

the $\sqrt{5}$ times itself is 5

$= \frac{3\sqrt{10}}{5}$

Example 10

Simplify $\frac{5\sqrt{3}}{\sqrt{11}}$

$\frac{5\sqrt{3}}{\sqrt{11}}$ multiply the top and bottom by $\sqrt{11}$

$= \frac{5\sqrt{3}}{\sqrt{11}} \bullet \frac{\sqrt{11}}{\sqrt{11}}$

$5\sqrt{3} \bullet \sqrt{11} = 5\sqrt{33}$

the $\sqrt{11}$ times itself is 11

$= \frac{5\sqrt{33}}{11}$

Example 13

Simplify $\frac{4\sqrt{5}}{\sqrt{6}}$

$\frac{4\sqrt{5}}{\sqrt{6}}$ multiply the top and bottom by $\sqrt{6}$

$= \frac{4\sqrt{5}}{\sqrt{6}} \bullet \frac{\sqrt{6}}{\sqrt{6}}$ $4\sqrt{5} \bullet \sqrt{6} = 4\sqrt{30}$
the $\sqrt{6}$ times itself is 6

$= \frac{4\sqrt{30}}{6}$ reduce $\frac{4}{6}$

$= \frac{2\sqrt{30}}{3}$

Example 14

Simplify $\frac{7\sqrt{10}}{\sqrt{6}}$

the 10 and 6 are both factors under a radical sign so they can reduce each other

$\frac{7\sqrt{5}}{\sqrt{3}}$ multiply the top and bottom by $\sqrt{3}$

$= \frac{7\sqrt{5}}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}}$ $7\sqrt{5} \bullet \sqrt{3} = 7\sqrt{15}$
the $\sqrt{3}$ times itself is 3

$= \frac{7\sqrt{15}}{3}$

