Section 9 – 4: Multiplying Radical Expressions

Multiplying a Monomial times a Monomial

To multiply **a monomial term times a monomial term** with radicals you use the following rule

$$\mathrm{A}\sqrt{\mathrm{B}} \bullet C\sqrt{D}$$
$$= A \bullet C \bullet \sqrt{B \bullet D}$$

In other words you **multiply the coefficients** of the radical terms times the **product of the terms under the radicals.**

Example 1

$(5\sqrt{3})(4\sqrt{7})$

$= 5 \bullet 4\sqrt{3 \bullet 7}$

$= 20\sqrt{21}$

Example 2

$(8\sqrt{5})(2\sqrt{7})$

$= 8 \bullet 2\sqrt{5 \bullet 7}$

$= 16\sqrt{35}$

Example 3

$(2\sqrt{3})(-3\sqrt{5})$

$= 2 \bullet (-3)\sqrt{3 \bullet 5}$

$= -6\sqrt{15}$

After you multiply the monomial terms you may be able to reduce the number under the radical sign. To reduce the number under the radical you factor it, looking for the largest **perfect square factor** or a **pair of factors** as we did in the first section. You then take out any factor that is a perfect square or any pairs of factors.

In these examples we will multiply the numbers under the radical together and then reduce by looking for the largest factor that is a perfect square. We then reduce the perfect square factor and simplify

Example 4

$(2\sqrt{3})(5\sqrt{6}) = 2 \bullet 5\sqrt{3 \bullet 6}$
$= 10\sqrt{18}$

reduce $10\sqrt{18}$

$= 10\sqrt{9 \bullet 2}$
$= 10\sqrt{9}\sqrt{2}$
$= 10 \bullet 3\sqrt{2}$
$= 30\sqrt{2}$

Example 5

$(2\sqrt{6})(3\sqrt{8}) = 2 \bullet 3\sqrt{6 \bullet 8}$
$= 6\sqrt{48}$

reduce $6\sqrt{48}$

$= 6\sqrt{16 \bullet 3}$
$= 6 \bullet \sqrt{16} \bullet \sqrt{3}$
$= 6 \bullet 4\sqrt{3}$
$= 24\sqrt{3}$

Example 6

$(3\sqrt{2})(5\sqrt{12}) = 3 \bullet 5\sqrt{2 \bullet 12}$
$= 15\sqrt{24}$

reduce $15\sqrt{24}$

$= 15\sqrt{4 \bullet 6}$
$= 15 \bullet \sqrt{4} \bullet \sqrt{6}$
$= 15 \bullet 2\sqrt{6}$
$= 30\sqrt{6}$

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If the numbers under the radical are large it may be faster to **factor each radicand before you multiply and list the factors under a single square root.**

$$\text{If } \sqrt{A} = \sqrt{C \bullet D}$$
$$\text{and } \sqrt{B} = \sqrt{E \bullet F}$$
$$\text{then } \sqrt{A} \bullet \sqrt{B} = \sqrt{C \bullet D \bullet E \bullet F}$$

Then reduce by taking out any pairs of factors or perfect squares.

In the examples below we will multiply the radicals by listing all the factors of each radicand as factors under one radical sign. This saves the work of getting a large number that needs to be factored back into the two factors we just multiplied together. We then reduce by taking out pairs of factors or perfect squares.

Example 7

$\left(\sqrt{20}\right)\left(\sqrt{6}\right)$

factor each radicand

$= \sqrt{4 \bullet 5 \bullet 2 \bullet 3}$

$= \sqrt{4} \bullet \sqrt{5 \bullet 2 \bullet 3}$

$= 2 \bullet \sqrt{5 \bullet 2 \bullet 3}$

$= 2\sqrt{30}$

Example 8

$\left(\sqrt{14}\right)\left(\sqrt{21}\right)$

factor each radicand

$= \sqrt{2 \bullet 7 \bullet 3 \bullet 7}$

$= \sqrt{7 \bullet 7} \bullet \sqrt{2 \bullet 3}$

$= 7 \bullet \sqrt{2 \bullet 3}$

$= 2\sqrt{6}$

Example 9

$\left(\sqrt{30}\right)\left(\sqrt{50}\right)$

factor each radicand

$= \sqrt{3 \bullet 10 \bullet 5 \bullet 10}$

$= \sqrt{10 \bullet 10} \bullet \sqrt{3 \bullet 5}$

$= 10 \bullet \sqrt{3 \bullet 5}$

$= 10\sqrt{15}$

Example 10

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$\left(2\sqrt{7}\right)\left(5\sqrt{14}\right)$

$= 2 \bullet 5\sqrt{7 \bullet 7 \bullet 2}$

reduce $10\sqrt{7 \bullet 7 \bullet 2}$

$= 10 \bullet \sqrt{7 \bullet 7} \bullet \sqrt{2}$

$= 10 \bullet 7\sqrt{2}$

$= 70\sqrt{2}$

Example 11

$\left(2\sqrt{10}\right)\left(4\sqrt{15}\right)$

$= 2 \bullet 4\sqrt{2 \bullet 5 \bullet 3 \bullet 5}$

reduce $8 \bullet \sqrt{2 \bullet 5 \bullet 3 \bullet 5}$

$= 8\sqrt{5 \bullet 5} \bullet \sqrt{2 \bullet 3}$

$= 8 \bullet 5 \bullet \sqrt{6}$

$= 40\sqrt{6}$

Example 12

$\left(\sqrt{15}\right)\left(4\sqrt{35}\right)$

$= 4\sqrt{3 \bullet 5 \bullet 7 \bullet 5}$

reduce $4\sqrt{3 \bullet 5 \bullet 7 \bullet 5}$

$= 4 \bullet \sqrt{5 \bullet 5}\sqrt{3 \bullet 7}$

$= 4 \bullet 5 \bullet \sqrt{21}$

$= 20\sqrt{21}$

Multiplying a Monomial times a Polynomial (Distributive Property)

The Distributive Rule With Radicals

A distributive operation has a **monomial term outside** a parenthesis and a **polynomial** expression **inside**. To distribute the monomial term you multiply each term inside the parentheses by the monomial term outside the parenthesis.

Example 1

$\sqrt{2}\left(3-\sqrt{5}\right)$

$= 3\bullet\sqrt{2}-\sqrt{2}\bullet\sqrt{5}$

$= 3\sqrt{2}-\sqrt{10}$

Example 2

$\sqrt{2}\left(4\sqrt{3}-\sqrt{5}\right)$

$= \sqrt{2}\bullet 4\sqrt{3}-\sqrt{2}\sqrt{5}$

$= 4\sqrt{6}-\sqrt{10}$

Example 3

$3\sqrt{5}\left(2-\sqrt{3}\right)$

$= 2\bullet 3\sqrt{5}-3\sqrt{5}\sqrt{3}$

$= 6\sqrt{5}-3\sqrt{15}$

It is common to be able to reduce one or more of the radicals after you distribute.

Example 4

$\sqrt{3}\left(4-5\sqrt{6}\right)$

distribute the $\sqrt{3}$

$= 4\sqrt{3}-5\sqrt{18}$

reduce $5\sqrt{18}$

$= 4\sqrt{3}-5\sqrt{9\bullet 2}$

$= 4\sqrt{3}-5\bullet\sqrt{9}\bullet\sqrt{2}$

$= 4\sqrt{3}-15\sqrt{2}$

Example 5

$\sqrt{5}\left(6+2\sqrt{10}\right)$

distribute the $\sqrt{5}$

$= 6\sqrt{5}+2\sqrt{50}$

reduce $2\sqrt{50}$

$= 6\sqrt{5}+2\sqrt{25\bullet 2}$

$= 6\sqrt{5}+2\bullet\sqrt{25}\bullet\sqrt{2}$

$= 6\sqrt{5}+10\sqrt{2}$

Example 6

$5\sqrt{2}\left(3\sqrt{6}+7\right)$

distribute the $5\sqrt{2}$

$= 15\sqrt{12}+35\sqrt{2}$

reduce $15\sqrt{12}$

$= 15\sqrt{4\bullet 3}+35\sqrt{2}$

$= 15\sqrt{4}\sqrt{3}+35\sqrt{2}$

$= 30\sqrt{3}+35\sqrt{2}$

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Example 7

$\sqrt{3}\left(\sqrt{3}+\sqrt{15}\right)$

distribute the $\sqrt{3}$

$=\sqrt{9}+\sqrt{45}$

reduce $\sqrt{9}\ =3$

reduce $\sqrt{45}=\sqrt{9\bullet 5}$

$=3+\sqrt{9\bullet 5}$

$=3+\sqrt{9}\sqrt{5}$

$=3+3\sqrt{5}$

Example 8

$\sqrt{6}\left(5\sqrt{3}+\sqrt{2}\right)$

distribute the $\sqrt{6}$

$=5\sqrt{18}+\sqrt{12}$

reduce $5\sqrt{18}\ =5\sqrt{9\bullet 2}$

reduce $\sqrt{12}=\sqrt{4\bullet 3}$

$=5\sqrt{9}\sqrt{2}\ +\sqrt{4}\sqrt{3}$

$=5\bullet 3\sqrt{2}\ +2\sqrt{3}$

$=15\sqrt{2}+2\sqrt{3}$

Example 9

$2\sqrt{7}\left(\sqrt{7}+3\sqrt{14}\right)$

distribute the $2\sqrt{7}$

$=2\sqrt{49}+6\sqrt{7\bullet 14}$

reduce $\sqrt{49}\ =7$

reduce $6\sqrt{7\bullet 14}=6\sqrt{7\bullet 7\bullet 2}$

$=7+6\bullet\sqrt{7\bullet 7}\sqrt{2}$

$=7+6\bullet 7\sqrt{2}$

$=7+42\sqrt{2}$

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Multiplying a Binomial times a Binomial (FOIL)

We have used the FOIL process in other chapters to find **the product of two binomials** where the terms of the binomials were variable terms. We will now consider how the product of two binomials will FOIL if some or all of the terms in the binomials contain radicals.

Multiply $\left(2-\sqrt{5}\right)\left(4-\sqrt{3}\right)$ **using FOIL**

F. O. I. L.

We distribute both of the terms in the first binomial to both of the terms in the second binomial using the following order

We use the mnemonic **F O I L** to help us remember the process

F The product of the two **First** terms $(2-\sqrt{5})(4-\sqrt{3})=8$

O The product of the two **Outer** terms $(2-\sqrt{5})(4-\sqrt{3})=-2\sqrt{3}$

I The product of the two **Inner** terms $(2-\sqrt{5})(4-\sqrt{3})=-4\sqrt{5}$

L The product of the two Last terms $(2-\sqrt{5})(4-\sqrt{3})=\sqrt{15}$

$(2-\sqrt{5})(4-\sqrt{3})=$

F O I L

$8-2\sqrt{3}-4\sqrt{5}+\sqrt{15}$

It is common to be able to reduce one or more of the radicals after you FOIL. It is very common to be able to combine Like Terms after you FOIL and reduce the radicals.

Example 1

$(5-\sqrt{3})(2-\sqrt{3})=$

F O I L

$=10-5\sqrt{3}-3\sqrt{3}+\sqrt{9}$

$=10-5\sqrt{3}-2\sqrt{3}+3$

$=13-7\sqrt{3}$

Example 2

$(3-\sqrt{7})(1+\sqrt{7})=$

F O I L

$=3+3\sqrt{7}-1\sqrt{7}+\sqrt{49}$

$=13+3\sqrt{7}-1\sqrt{7}-7$

$=6+2\sqrt{7}$

Example 3

$(1-\sqrt{5})(2+\sqrt{5})=$

F O I L

$=2+1\sqrt{5}-2\sqrt{5}-\sqrt{25}$

$=2+1\sqrt{5}-2\sqrt{5}-5$

$=-3-\sqrt{5}$

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Sometimes the FOIL process reduces the product to a single integer. This is a very special case. This happens when one of the binomials is the sum of two terms and the other binomial is the difference of the two terms. We call this combination the **Square Root Conjugates**. We will use these **Square Root Conjugates** in the next section.

Foiling Square Root Conjugates produces a rational number

$$\left(A+\sqrt{B}\right)\left(A-\sqrt{B}\right)=$$

F O I L

$$=A^2-A\sqrt{B}+A\sqrt{B}-\sqrt{B\bullet B}$$
$$=A^2-\sqrt{B\bullet B}$$
$$=A^2-B$$

$$\left(\sqrt{A}+\sqrt{B}\right)\left(\sqrt{A}-\sqrt{B}\right)=$$

F O I L

$$=\sqrt{A\bullet A}-\sqrt{A\bullet B}+\sqrt{A\bullet B}-\sqrt{B\bullet B}$$
$$=\sqrt{A\bullet A}-\sqrt{B\bullet B}$$
$$=A-B$$

Example 1

$$(5-\sqrt{3})(5+\sqrt{3})=$$

F O I L

$$=25+5\sqrt{3}-5\sqrt{3}-\sqrt{9}$$
$$=25-3$$
$$=22$$

Example 2

$$(\sqrt{6}+4)(\sqrt{6}-4)=$$

F O I L

$$=\sqrt{36}-4\sqrt{6}+4\sqrt{6}-16$$
$$=6-16$$
$$=-10$$

Example 3

$$(7-3\sqrt{2})(7+3\sqrt{2})=$$

F O I L

$$=49+21\sqrt{2}-21\sqrt{2}-9\sqrt{4}$$
$$=49-18$$
$$=31$$

Example 4

$$(2\sqrt{3}+5)(2\sqrt{3}-5)=$$

F O I L

$$=4\sqrt{9}-10\sqrt{3}+10\sqrt{3}-25$$
$$=12-25$$
$$=-13$$

Example 5

$$(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})=$$

F O I L

$$=\sqrt{9}-\sqrt{15}+\sqrt{15}-\sqrt{25}$$
$$=3-5$$
$$=-2$$

Example 6

$$(\sqrt{7}-\sqrt{2})(\sqrt{7}+\sqrt{2})=$$

F O I L

$$=\sqrt{49}+\sqrt{14}-\sqrt{14}-\sqrt{4}$$
$$=7-2$$
$$=5$$