

## Section 9 – 3: Combining (Adding or Subtracting) Radical Expressions

### Like Terms

When we combined polynomials in Chapter 1 we were only able to combine like terms. In a polynomial the like terms were the terms with the exact same variable names. This same restriction is true when we combine terms with radicals. You can only combine radical terms if the radical parts of the terms are alike. Radical terms are alike if the expression under the radicals is **exactly the same**.

#### Example of radical terms that ARE alike:

$$3\sqrt{5} + 4\sqrt{5}$$

the  $\sqrt{5}$  in both terms are exactly the same  
so the terms CAN be combined

$$2\sqrt{3x} - \sqrt{3x}$$

the  $\sqrt{3x}$  in both terms are exactly the same  
so the terms CAN be combined

#### Example of radical terms that are NOT alike:

$$6\sqrt{2} - 7\sqrt{3}$$

the  $\sqrt{2}$  and the  $\sqrt{3}$  are NOT exactly the same  
so the terms CAN NOT be combined

$$6\sqrt{2xy} - 7\sqrt{2x}$$

the  $\sqrt{2xy}$  and the  $\sqrt{2x}$  are NOT exactly the same  
so the terms CAN NOT be combined

### Combining Like Terms

Combining like terms involves determining the **total of the coefficients** of all the like terms. This is done by adding or subtracting the coefficients of the like terms and using that number as the coefficient of the like term.

#### To Combine Like Terms

1. Combine the coefficients of the like terms by **adding or subtracting the coefficients**.
2. Using the sum of the coefficients as the coefficient of common radical expression.

#### Example 1

$$3\sqrt{5} + 4\sqrt{5}$$

the  $\sqrt{5}$  in both terms are  
exactly the same so  
the terms CAN be combined

$$\begin{aligned} &\text{combine the 3 and 4} \\ &= 7\sqrt{5} \end{aligned}$$

#### Example 2

$$4\sqrt{3} + 2\sqrt{3} - 9\sqrt{3}$$

the  $\sqrt{3}$  in all the terms are  
exactly the same so  
the terms CAN be combined

$$\begin{aligned} &\text{combine the 4, 2 and } -9 \\ &= -3\sqrt{3} \end{aligned}$$

#### Example 3

$$-6\sqrt{7} - \sqrt{7} + 3\sqrt{7}$$

the  $\sqrt{7}$  in all the terms are  
exactly the same so  
the terms CAN be combined

$$\begin{aligned} &\text{combine the } -6, -1 \text{ and } 3 \\ &= -4\sqrt{7} \end{aligned}$$

Sometimes only some of the terms in an expression are alike. In that case you can only combine the terms that are alike.

#### Example 4

$$3\sqrt{6} - 7\sqrt{2} + 5\sqrt{6}$$

only the  $3\sqrt{6}$  and  $5\sqrt{6}$  are like terms  
combine only the 3 and the 5

$$= 8\sqrt{6} - 7\sqrt{2}$$

#### Example 6

$$2\sqrt{3} + 8\sqrt{2} - 9\sqrt{3} - 5\sqrt{2}$$

the  $2\sqrt{3}$  and  $-9\sqrt{3}$  are like terms  
combine the 2 and the  $-9$

$$-7\sqrt{3}$$

the  $+8\sqrt{2}$  and  $-5\sqrt{2}$  are like terms  
combine the 8 and the  $-5$

$$3\sqrt{2}$$

$$= -7\sqrt{3} - 3\sqrt{2}$$

#### Example 5

$$-2\sqrt{11} + 6\sqrt{11} - 3\sqrt{2}$$

only the  $-2\sqrt{11}$  and  $+6\sqrt{11}$  are like terms  
combine only the  $-2$  and the 6

$$= 4\sqrt{11} - 3\sqrt{2}$$

#### Example 7

$$7\sqrt{11} + 3\sqrt{7} - 8\sqrt{11} - 11\sqrt{7}$$

the  $7\sqrt{11}$  and  $-8\sqrt{11}$  are like terms  
combine the 7 and the  $-8$

$$-\sqrt{11}$$

the  $+3\sqrt{7}$  and  $-11\sqrt{7}$  are like terms  
combine the 3 and the  $-11$

$$-8\sqrt{7}$$

$$= -\sqrt{11} - 8\sqrt{7}$$

## Simplifying Radical Expressions and then Combining Like Terms

Some radical expressions do not appear to have like terms that can be combined but after the radicals are simplified there are like terms that can be combined.

### Example 8

$$5\sqrt{7} - \sqrt{28}$$

$\sqrt{7}$  and  $\sqrt{28}$  are NOT like terms  
but the  $\sqrt{28}$  can be reduced

$$-\sqrt{28} = -\sqrt{4 \cdot 7} = -2\sqrt{7}$$

$$\text{so } 5\sqrt{7} - \sqrt{28}$$

$$= 5\sqrt{7} - 2\sqrt{7}$$

which can be combined

$$= 3\sqrt{7}$$

### Example 10

$$8\sqrt{20} - \sqrt{45}$$

The  $\sqrt{20}$  and  $\sqrt{45}$  are NOT like terms  
but they can be reduced

$$8\sqrt{20} = 8\sqrt{4 \cdot 5} = 8(2)\sqrt{5} = 16\sqrt{5}$$

$$-\sqrt{45} = -\sqrt{9 \cdot 5} = -(3)\sqrt{5} = -3\sqrt{5}$$

$$\text{so } 8\sqrt{20} - \sqrt{45}$$

$$= 16\sqrt{5} - 3\sqrt{5}$$

which can be combined

$$= 13\sqrt{5}$$

### Example 9

$$-2\sqrt{18} - 5\sqrt{2}$$

$\sqrt{18}$  and  $\sqrt{2}$  are NOT like terms  
but the  $\sqrt{18}$  can be reduced

$$-2\sqrt{18} = -2\sqrt{9 \cdot 2} = -2(3)\sqrt{2}$$

$$\text{so } -2\sqrt{18} - 5\sqrt{2}$$

$$= -6\sqrt{2} - 5\sqrt{2}$$

which can be combined

$$= -11\sqrt{2}$$

### Example 11

$$-2\sqrt{18} + 5\sqrt{2}$$

$\sqrt{18}$  and  $\sqrt{2}$  are NOT like terms  
but the  $\sqrt{18}$  can be reduced

$$-2\sqrt{18} = -2\sqrt{9 \cdot 2} = -2(3)\sqrt{2}$$

$$\text{so } -2\sqrt{18} + 5\sqrt{2}$$

$$= -6\sqrt{2} + 5\sqrt{2}$$

which can be combined

$$= -\sqrt{2}$$

**Example 12**

$$2\sqrt{48} + 4\sqrt{75}$$

The  $\sqrt{48}$  and  $\sqrt{75}$  are NOT like terms but they can be reduced

$$2\sqrt{48} = 2\sqrt{16 \cdot 3} = 2(4)\sqrt{3} = 8\sqrt{3}$$

$$+4\sqrt{75} = 4\sqrt{25 \cdot 3} = 4(5)\sqrt{3} = 20\sqrt{3}$$

$$\text{so } 2\sqrt{48} + 4\sqrt{75}$$

$$= 8\sqrt{3} + 20\sqrt{3}$$

which can be combined

$$= 28\sqrt{3}$$

**Example 13**

$$-7\sqrt{8} + 3\sqrt{50}$$

The  $\sqrt{8}$  and  $\sqrt{50}$  are NOT like terms but they can be reduced

$$-7\sqrt{8} = -7\sqrt{4 \cdot 2} = -7(2)\sqrt{2} = -14\sqrt{2}$$

$$3\sqrt{50} = 3\sqrt{25 \cdot 2} = 3(5)\sqrt{2} = 15\sqrt{2}$$

$$\text{so } -7\sqrt{8} + 3\sqrt{50}$$

$$= -14\sqrt{2} + 15\sqrt{2}$$

which can be combined

$$= \sqrt{2}$$

Sometimes even after you simplify each radical term some of the terms in an expression are alike and others are not. In that case you can only combine the terms that are alike.

**Example 14**

$$2\sqrt{27} - \sqrt{50} + 5\sqrt{12}$$

$$2\sqrt{27} = 2\sqrt{9 \cdot 3} = 2(3)\sqrt{3} = 6\sqrt{3}$$

$$-\sqrt{50} = -\sqrt{25 \cdot 2} = -(5)\sqrt{2} = -5\sqrt{2}$$

$$+5\sqrt{12} = 5\sqrt{4 \cdot 3} = 5(2)\sqrt{3} = 10\sqrt{3}$$

$$= 6\sqrt{3} - 5\sqrt{2} + 10\sqrt{3}$$

$$= 16\sqrt{3} - 5\sqrt{2}$$

**Example 15**

$$4\sqrt{18} + 3\sqrt{20} - 5\sqrt{32}$$

$$4\sqrt{18} = 4\sqrt{9 \cdot 2} = 4(3)\sqrt{2} = 12\sqrt{2}$$

$$+3\sqrt{20} = 3\sqrt{4 \cdot 5} = 3(2)\sqrt{5} = 6\sqrt{5}$$

$$-5\sqrt{32} = -5\sqrt{16 \cdot 2} = -5(4)\sqrt{2} = -20\sqrt{2}$$

$$= 12\sqrt{2} + 6\sqrt{5} - 20\sqrt{2}$$

$$= -8\sqrt{2} + 6\sqrt{5}$$