

## Section 9 – 2B: Simplifying Radical Expressions

### Simplifying Radicals with Variable Factors

The last section introduced the concept of reducing radicals by taking out pairs of a common factor. Two of the same factors under a square root form a perfect square. This means that if you have a pair of the same factors under a square root they can be reduced to a rational number.

$$\sqrt{4} = \sqrt{2 \cdot 2} = 2$$

a pair of 2's under  
a square root reduce to  
the whole number 2

$$\sqrt{9} = \sqrt{3 \cdot 3} = 3$$

a pair of 3's under  
a square root reduce to  
the whole number 3

$$\sqrt{81} = \sqrt{3 \cdot 3 \cdot 3 \cdot 3}$$

$$= \sqrt{3 \cdot 3} \cdot \sqrt{3 \cdot 3}$$

$$= 3 \cdot 3 = 9$$

2 pair of 3's under  
a square root reduce to  
the whole number 9

This fact also allows us to reduce radical expressions with terms that contain variables as factors under the square root symbol. Completely factor the expression under the square root into its many factors and then take out the pairs of same factors. **Assume that all variables represent positive numbers.**

$$\sqrt{a^2} = \sqrt{a \cdot a} = a$$

where a is a positive number

#### Example 1

$$\sqrt{a^7}$$

completely factor  $a^7$

$$= \sqrt{a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a}$$

put pairs of the same factor  
under their own square root

$$= \sqrt{a \cdot a} \cdot \sqrt{a \cdot a} \cdot \sqrt{a \cdot a} \cdot \sqrt{a}$$

each of the 3 pairs of a's  
can be reduced to an a

$$= a \cdot a \cdot a \cdot \sqrt{a}$$

$$= a^3 \sqrt{a}$$

#### Example 2

$$\sqrt{a^6}$$

completely factor  $a^6$

$$= \sqrt{a \cdot a \cdot a \cdot a \cdot a \cdot a}$$

put pairs of the same factor  
under their own square root

$$= \sqrt{a \cdot a} \cdot \sqrt{a \cdot a} \cdot \sqrt{a \cdot a}$$

each of the 2 pairs of a's  
can be reduced to an a

$$= a \cdot a \cdot a$$

$$= a^3$$

## Save Time by Determining the Number Of Pairs

We can save some work by determining **how many pairs of factors a variable under the square root symbol has**. Write each pair under its own square root. Reduce each of these square roots. If the exponent of the variable under the square is **odd** then a single power of that variable will remain under the square root. If the exponent of the variable is **even** then that variable will not appear under the square root.

### Example 1

Simplify:  $\sqrt{y^6}$

$y^6$  has 3 pairs of  $y^2$   
with none left over so

$$\sqrt{y^6} = \sqrt{y^2} \cdot \sqrt{y^2} \cdot \sqrt{y^2}$$

$$\sqrt{y^6} = y \cdot y \cdot y$$

$$\sqrt{y^6} = y^3$$

### Example 2

$$\sqrt{x^7}$$

$x^7$  has 3 pairs of  $x^2$   
with 1  $x$  left over so

$$\sqrt{x^7} = \sqrt{x^2} \cdot \sqrt{x^2} \cdot \sqrt{x^2} \cdot \sqrt{x}$$

$$\sqrt{x^7} = x \cdot x \cdot x \cdot \sqrt{x}$$

$$\sqrt{x^7} = x^3 \sqrt{x}$$

### Example 3

$$\sqrt{x^8}$$

$x^8$  has 4 pairs of  $x^2$   
with none left over so

$$\sqrt{x^8} = \sqrt{x^2} \cdot \sqrt{x^2} \cdot \sqrt{x^2} \cdot \sqrt{x^2}$$

$$\sqrt{x^8} = x \cdot x \cdot x \cdot x$$

$$\sqrt{x^8} = x^4$$

We can eliminate writing the separate pairs of square roots and simplify each variable in one step by asking how many pairs does the variable have and is one left over or not.

### Example 4

Simplify:  $\sqrt{y^7}$

$y^7$  has 3 pairs of  $y^2$   
with 1  $y$  left over

$$\sqrt{y^7} = y^3 \sqrt{y}$$

### Example 5

Simplify:  $\sqrt{x^6}$

$x^6$  has 3 pairs of  $x^2$   
with no  $x$  left over

$$\sqrt{x^6} = x^3$$

### Example 6

Simplify:  $\sqrt{w^{11}}$

$w^{11}$  has 5 pairs of  $w^2$   
with 1  $w$  left over

$$\sqrt{w^{11}} = w^5 \sqrt{w}$$

**Example 7**

$$\sqrt{x^2 y^5 w^3}$$

$$\sqrt{x^2} = x$$

$$\sqrt{y^5} = y^2 \sqrt{y}$$

$$\sqrt{w^3} = w \sqrt{w}$$

$$\sqrt{x^2 y^5 w^3} = xy^2 w \sqrt{yw}$$

**Example 9**

$$\sqrt{x^5 y^6 w^8}$$

$$\sqrt{x^5} = x^2 \sqrt{x}$$

$$\sqrt{y^6} = y^3$$

$$\sqrt{w^8} = w^4$$

$$\sqrt{x^5 y^6 w^8} = x^2 y^3 w^4 \sqrt{x}$$

**Example 8**

$$\sqrt{x^{12} y^8 w^7}$$

$$\sqrt{x^{12}} = x^6$$

$$\sqrt{y^8} = y^4$$

$$\sqrt{w^7} = w^3 \sqrt{w}$$

$$\sqrt{x^{12} y^8 w^7} = x^6 y^4 w^3 \sqrt{w}$$

**Example 10**

$$\sqrt{x^5 y^7 w^3}$$

$$\sqrt{x^5} = x^2 \sqrt{x}$$

$$\sqrt{y^7} = y^3 \sqrt{y}$$

$$\sqrt{w^3} = w \sqrt{w}$$

$$\sqrt{x^5 y^7 w^3} = x^2 y^3 w \sqrt{xyw}$$

## Putting it all together

It is common that the factors under the square root will contain **the product of a number and one or more variables**. To reduce these radicals reduce the coefficient of the variables by looking for perfect square factors or pairs of factors. Then reduce each variable by asking how many pairs of that variable there are and is one left over that will remain under the square root.

### Example 1

$$\sqrt{12x^5y^6}$$

$$\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$$

$$\sqrt{x^5} = x^2\sqrt{x}$$

$$\sqrt{y^6} = y^3$$

$$\sqrt{12x^5y^6} = 2x^2y^3\sqrt{3x}$$

### Example 2

$$\sqrt{18x^8y^{10}}$$

$$\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$$

$$\sqrt{x^8} = x^4$$

$$\sqrt{y^{10}} = y^5$$

$$\sqrt{18x^8y^{10}} = 3x^4y^5\sqrt{2}$$

### Example 3

$$\sqrt{36x^4y^3}$$

$$\sqrt{36} = 6$$

$$\sqrt{x^4} = x^2$$

$$\sqrt{y^3} = y\sqrt{y}$$

$$\sqrt{36x^4y^3} = 6x^2y\sqrt{y}$$

### Example 4

$$\sqrt{44x^4y^9}$$

$$\sqrt{44} = \sqrt{4 \cdot 11} = 2\sqrt{11}$$

$$\sqrt{x^4} = x^2$$

$$\sqrt{y^9} = y^4\sqrt{y}$$

$$\sqrt{44x^4y^9} = 2x^2y^4\sqrt{11y}$$

### Example 5

$$\sqrt{49x^2y^{12}}$$

$$\sqrt{49} = 7$$

$$\sqrt{x^2} = x$$

$$\sqrt{y^{12}} = y^6$$

$$\sqrt{49x^2y^{12}} = 7xy^6$$

### Example 6

$$\sqrt{63x^3y^{11}}$$

$$\sqrt{63} = \sqrt{9 \cdot 7} = 3\sqrt{7}$$

$$\sqrt{x^3} = x\sqrt{x}$$

$$\sqrt{y^{11}} = y^5\sqrt{y}$$

$$\sqrt{63x^3y^{11}} = 3xy^5\sqrt{7xy}$$