Section 9 – 2A: Simplifying Radical Expressions

Rational Numbers

A Rational Number is any number that that can be expressed as

a whole number	**a fraction**	**a decimal that ends**	**a decimal that repeats**
3	$\frac{7}{2}$	1.2	$1.\overline{333}$

In the last section every number under the square root symbol was a **perfect square**. The square root of a perfect square can be replaced with a rational number.

$\sqrt{9}$ can be replaced with a 3	$\sqrt{\frac{49}{4}}$ can be replaced with a $\frac{7}{2}$	$\sqrt{\frac{36}{25}}$ can be replaced with a $\frac{6}{5}$ or 1.2	$\sqrt{\frac{16}{9}}$ can be replaced with a $\frac{4}{3}$ $1.\overline{333}$

Irrational numbers

Irrational Numbers are decimal numbers whose digits go on and on without ending and the digits never repeat in any pattern. Irrational Numbers are numbers that **CANNOT BE EXPRESSED** as a **whole number**, a **fraction,** a **decimal that ends** or a **decimal that repeats.**

The square root of a number that is not a perfect square is **an irrational number.** You cannot write a decimal that does not end or repeat. This means that **the square root of numbers that are not perfect squares cannot be written without a radical sign.**

$\sqrt{7}$ is an irrational number. It cannot be replaced with a whole number, a decimal or fraction. It can only be written as $\sqrt{7}$	$\sqrt{21}$ is an irrational number. It cannot be replaced with a whole number, a decimal or fraction. It can only be written as $\sqrt{21}$	$\sqrt{13}$ is an irrational number. It cannot be replaced with a whole number, a decimal or fraction. It can only be written as $\sqrt{13}$

Radicands

The number under the square is called the **radicand.**

There are 3 types of radicands.

The radicand is a Perfect Square

$$\sqrt{\text{the radicand is a perfect square}}$$

These square roots **can be reduced to rational numbers**. The answer will NOT have a radical sign.

Example 1

$\sqrt{9}$

can be replaced with 3

$\sqrt{9} = 3$

Example 2

$\sqrt{\frac{49}{16}}$

can be replaced with 7 / 4

$\sqrt{\frac{49}{16}} = \frac{7}{4}$

The radicand DOES NOT have a factor that is a Perfect Square

$$\sqrt{\text{the radicand DOES NOT have a factor that is a perfect square}}$$

These square roots **CANNOT BE REDUCED at all**.

Example 5

$\sqrt{7} = \sqrt{1 \bullet 7}$

No factor is a Perfect Square

It can only be written as

$\sqrt{7}$

Example 6

$\sqrt{34} = \sqrt{2 \bullet 17}$

No factor is a Perfect Square

It can only be written as

$\sqrt{34}$

The radicand has a factor that is a Perfect Square

$$\sqrt{\text{the radicand has a factor that is a perfect square}}$$

If one of the factors **is a perfect square** then the square root can be reduced to a rational number times a square root of an irrational number smaller than the original square root you started with. **The answer will still have a square root in it.**

Example 3

$\sqrt{8} = \sqrt{4 \bullet 2}$

can be replaced by

$2\sqrt{2}$

Example 4

$\sqrt{75} = \sqrt{25 \bullet 3}$

can be replaced by

$5\sqrt{3}$

Reducing a Square Root Expression

If the radicand is an irrational number that has a **factor** that is a perfect square then the square root can be reduced to a rational number outside the radical sign times the square root of an irrational number smaller than the original radicand you started with. This process is called reducing or simplifying the square root. Reducing a square root requires the use of the Multiplication Rule for Square Roots.

The Multiplication Rule for Square Roots

$$\sqrt{C} = \sqrt{A \bullet B} = \sqrt{A} \bullet \sqrt{B}$$

The Multiplication Rule for Square Roots allows you to **factor the number under the square root** and write the positive factors as a product with **each factor under a separate square root.**

$$\sqrt{15} = \sqrt{3 \bullet 5} = \sqrt{3} \bullet \sqrt{5} \qquad \sqrt{63} = \sqrt{9 \bullet 7} = \sqrt{9} \bullet \sqrt{7}$$

Simplifying Square Roots using PERFECT SQUARE FACTORS

This process requires that you find the **largest perfect square that is a factor of the original radicand.** Use the list of perfect squares below as a guide for finding the largest perfect square that is a factor of the original radicand for radicands whose value is from 1 to 225.

1	4	9	16	25	36	49	64	81
100	121	144	169	196	225			

Look for the **largest perfect square** in the list above that is a factor of the radicand. Factor the radicand and put each factor under a separate square root. **Reduce the square root of the perfect square** and **multiply that number times the remaining square root.**

Example 1

Simplify: $\sqrt{63}$

Factor using a 9

$= \sqrt{9 \bullet 7}$

seperate the square roots

$= \sqrt{9} \bullet \sqrt{7}$

reduce the perfect square factor

$= 3\sqrt{7}$

Example 2

Simplify: $\sqrt{12}$

Factor using a 4

$= \sqrt{4 \bullet 3}$

seperate the square roots

$= \sqrt{4} \bullet \sqrt{3}$

reduce the perfect square factor

$= 2\sqrt{3}$

Example 3

Simplify: $\sqrt{44}$

Factor using a 4

$= \sqrt{4 \bullet 11}$

seperate the square roots

$= \sqrt{4} \bullet \sqrt{11}$

reduce the perfect square factor

$= 2\sqrt{11}$

Example 4

Simplify: $\sqrt{125}$

Factor using a 25

$= \sqrt{25 \bullet 5}$

seperate the square roots

$= \sqrt{25} \bullet \sqrt{5}$

reduce the perfect square factor

$= 5\sqrt{5}$

Example 5

Simplify: $\sqrt{18}$

Factor using a 9

$= \sqrt{9 \bullet 2}$

seperate the square roots

$= \sqrt{9} \bullet \sqrt{2}$

reduce the perfect square factor

$= 3\sqrt{2}$

Example 6

Simplify: $\sqrt{48}$

Factor using a 16

$= \sqrt{16 \bullet 3}$

seperate the square roots

$= \sqrt{16} \bullet \sqrt{3}$

reduce the perfect square factor

$= 4\sqrt{3}$

Example 5 Note: To use this technique you must factor $\sqrt{18}$ as $\sqrt{9 \bullet 2}$ and not as $\sqrt{3 \bullet 6}$

Example 6 Note: To use this technique you must factor $\sqrt{48}$ as $\sqrt{16 \bullet 3}$ and not as $\sqrt{4 \bullet 12}$

Example 7

Simplify: $\sqrt{72}$

Factor using a 36

$= \sqrt{36 \bullet 2}$

seperate the square roots

$= \sqrt{36} \bullet \sqrt{2}$

reduce the perfect square factor

$= 6\sqrt{2}$

Example 8

Simplify: $\sqrt{80}$

Factor using a 16

$= \sqrt{16 \bullet 5}$

seperate the square roots

$= \sqrt{16} \bullet \sqrt{5}$

reduce the perfect square factor

$= 4\sqrt{5}$

Example 7 Note: To use this technique you must factor $\sqrt{72}$ as $\sqrt{36 \bullet 2}$ and not as $\sqrt{9 \bullet 8}$

Simplifying Square Root Radicals using PAIRS OF FACTORS

If you can find the largest perfect square factor of the radicand then reducing the radical expression is a short process. Many students cannot find the largest perfect square factor or they do not want to take the extended time this may take. There is an alternate approach that is favored by many students.

A pair (two) of the same factors under a square root form a perfect square. This means that if you have a pair of the same factors under a square root they can be reduced to a rational number.

$\sqrt{2 \bullet 2} = \sqrt{4} = 2$
a pair of 2's under
a square root reduce to
the whole number 2

$\sqrt{3 \bullet 3} = \sqrt{9} = 3$
a pair of 3's under
a square root reduce to
the whole number 3

$\sqrt{5 \bullet 5} = \sqrt{25} = 5$
a pair of 5's under
a square root reduce to
the whole number 5

This fact allows us to use the Multiplication Rule for Square Roots to completely factor a radicand into its many factors and then take out **any pairs of the same factors.**

Completely factor the radicand and take out any pairs of the same factors.

Example 1

Simplify: $\sqrt{24}$
completly factor 24

$= \sqrt{2 \bullet 2 \bullet 2 \bullet 3}$
put pairs of the same factor
under thier own square root

$= \sqrt{2 \bullet 2} \bullet \sqrt{2 \bullet 3}$
the pair of 2's can reduced
$= 2 \bullet \sqrt{6}$

Example 2

Simplify: $\sqrt{54}$
completly factor 54

$= \sqrt{3 \bullet 3 \bullet 3 \bullet 2}$
put pairs of the same factor
under thier own square root

$= \sqrt{3 \bullet 3} \bullet \sqrt{3 \bullet 2}$
the pair of 3's can reduced
$= 3 \bullet \sqrt{6}$

Example 3

Simplify: $\sqrt{48}$
completly factor 48

$= \sqrt{2 \bullet 2 \bullet 2 \bullet 2 \bullet 3}$
put pairs of the same factor
under thier own square root

$= \sqrt{2 \bullet 2} \bullet \sqrt{2 \bullet 2} \bullet \sqrt{3}$
each pair of 2's can reduced
$= 2 \bullet 2 \bullet \sqrt{3} = 4\sqrt{3}$

Example 4

Simplify: $\sqrt{18}$
$= \sqrt{6 \bullet 3}$
$= \sqrt{2 \bullet 3 \bullet 3}$
$= \sqrt{3 \bullet 3} \bullet \sqrt{2}$
$= 3\sqrt{2}$

Example 5

Simplify: $\sqrt{60}$
$= \sqrt{6 \bullet 10}$
$\sqrt{2 \bullet 3 \bullet 2 \bullet 5}$
$= \sqrt{2 \bullet 2} \bullet \sqrt{3 \bullet 5}$
$= 2\sqrt{15}$

Example 6

Simplify: $\sqrt{50}$
$= \sqrt{5 \bullet 10}$
$= \sqrt{5 \bullet 2 \bullet 5}$
$= \sqrt{5 \bullet 5} \bullet \sqrt{2}$
$= 5\sqrt{2}$

Example 7

Simplify: $\sqrt{72}$

$= \sqrt{12 \bullet 6}$

$\sqrt{2 \bullet 2 \bullet 3 \bullet 3 \bullet 2}$

$= \sqrt{2 \bullet 2}\sqrt{3 \bullet 3} \bullet \sqrt{2}$

$= 2 \bullet 3 \bullet \sqrt{2}$

$= 6\sqrt{2}$

Example 8

$\sqrt{80}$

$= \sqrt{8 \bullet 10}$

$\sqrt{2 \bullet 2 \bullet 2 \bullet 2 \bullet 5}$

$= \sqrt{2 \bullet 2}\sqrt{2 \bullet 2} \bullet \sqrt{5}$

$= 2 \bullet 2 \bullet \sqrt{5}$

$= 4\sqrt{5}$

Example 9

$\sqrt{120}$

$= \sqrt{12 \bullet 10}$

$= \sqrt{2 \bullet 6 \bullet 2 \bullet 5}$

$= \sqrt{2 \bullet 2 \bullet 3 \bullet 2 \bullet 5}$

$= \sqrt{2 \bullet 2} \bullet \sqrt{3 \bullet 2 \bullet 5}$

$= 2\sqrt{30}$

Simplifying Square Roots using both Pairs of Factors and Perfect Squares

Factor the radicand. If a factor is a perfect square do not factor it further. Take out all the pairs and Perfect Squares:

Example 10

Simplify: $\sqrt{72}$

$= \sqrt{9 \bullet 8}$

$\sqrt{9 \bullet 4 \bullet 2}$

$= \sqrt{9} \bullet \sqrt{4} \bullet \sqrt{2}$

$= 3 \bullet 2 \bullet \sqrt{2}$

$= 6\sqrt{2}$

Example 11

Simplify: $\sqrt{80}$

$= \sqrt{4 \bullet 20}$

$= \sqrt{4 \bullet 4 \bullet 5}$

$= \sqrt{4 \bullet 4} \bullet \sqrt{5}$

$= 4 \bullet \sqrt{5}$

$= 4\sqrt{5}$

Example 12

Simplify: $\sqrt{120}$

$= \sqrt{40 \bullet 3}$

$= \sqrt{4 \bullet 10 \bullet 3}$

$= \sqrt{4 \bullet 2 \bullet 5 \bullet 3}$

$= \sqrt{4} \bullet \sqrt{2 \bullet 5 \bullet 3}$

$= 2\sqrt{30}$

Example 13

Simplify: $\sqrt{72}$

$= \sqrt{12 \bullet 6}$

$\sqrt{3 \bullet 4 \bullet 3 \bullet 2}$

$= \sqrt{4} \bullet \sqrt{3 \bullet 3} \bullet \sqrt{2}$

$= 2 \bullet 3 \bullet \sqrt{2}$

$= 6\sqrt{2}$

Example 14

Simplify: $\sqrt{80}$

$= \sqrt{4 \bullet 20}$

$= \sqrt{4 \bullet 2 \bullet 10}$

$= \sqrt{4 \bullet 2 \bullet 2 \bullet 5}$

$= \sqrt{4} \bullet \sqrt{2 \bullet 2} \bullet \sqrt{5}$

$= 2 \bullet 2 \bullet \sqrt{5}$

$= 4\sqrt{5}$

Example 15

Simplify: $\sqrt{180}$

$= \sqrt{10 \bullet 18}$

$\sqrt{2 \bullet 5 \bullet 9 \bullet 2}$

$= \sqrt{9} \bullet \sqrt{2 \bullet 2} \bullet \sqrt{5}$

$= 3 \bullet 2 \bullet \sqrt{5}$

$= 6\sqrt{5}$

