Section 9 – 1: Perfect Squares

Perfect Squares and Square Roots

What numbers are Perfect Squares?

When a **positive number** is **multiplied by itself** the number obtained is called a **Perfect Square**.

$1 \bullet 1 = \boxed{1}$ $2 \bullet 2 = \boxed{4}$ $3 \bullet 3 = \boxed{9}$ $4 \bullet 4 = \boxed{16}$ $5 \bullet 5 = \boxed{25}$

$6 \bullet 6 = \boxed{36}$ $7 \bullet 7 = \boxed{49}$ $8 \bullet 8 = \boxed{64}$ $9 \bullet 9 = \boxed{81}$ $10 \bullet 10 = \boxed{100}$

$11 \bullet 11 = \boxed{121}$ $12 \bullet 12 = \boxed{144}$ $13 \bullet 13 = \boxed{169}$ $14 \bullet 14 = \boxed{196}$ $15 \bullet 15 = \boxed{225}$

Not all positive whole numbers are perfect squares.

30 is not a perfect square because there is not a positive number times itself that equals 30. 45 is also not a perfect square because there is no positive number times itself that equals 45. The numbers in the squares above are the only **whole numbers** from 1 to 225 that are perfect squares. All the other whole numbers from 1 to 225 are not perfect squares.

Some fractions are perfect squares. When a fraction is **multiplied by itself** the fraction obtained will be a **Perfect Square.**

$\frac{2}{3} \bullet \frac{2}{3} = \boxed{\frac{4}{9}}$ $\frac{5}{4} \bullet \frac{5}{4} = \boxed{\frac{25}{16}}$ $\frac{9}{8} \bullet \frac{9}{8} = \boxed{\frac{81}{64}}$ $\frac{1}{10} \bullet \frac{1}{10} = \boxed{\frac{1}{100}}$

$\frac{12}{11} \bullet \frac{12}{11} = \boxed{\frac{144}{121}}$ $\frac{3}{7} \bullet \frac{3}{7} = \boxed{\frac{9}{49}}$ $\frac{6}{13} \bullet \frac{6}{13} = \boxed{\frac{36}{169}}$ $\frac{14}{15} \bullet \frac{14}{15} = \boxed{\frac{196}{225}}$

Some fractions would be perfect squares if they were reduced.

$\frac{8}{18}$ does not seem to be a perfect square. When it is reduced $\frac{8}{18} = \frac{4}{9}$ is easy to see that the reduced form is a perfect square. The reduced form of $\frac{32}{50}$ is also a perfect square because $\frac{32}{50}$ reduces to $\frac{16}{25}$ which is a perfect square.

Finding the Square Root of a Perfect Square

The symbol $\sqrt{}$ is called a radical sign but it is also commonly called a **square root sign**. The square root of a number is written as $\sqrt{a}$ **where a is a positive number.**

$\sqrt{a}$ is read as "the square root of a"

$\sqrt{9}$ is read as "the square root of 9

$\sqrt{25}$ is read as "the square root of 25

$\sqrt{a}$ asks "What **positive number times itself is equal to a?**

$\sqrt{36}$ asks "What positive number **times itself** equals 36. Since $6 \bullet 6 = 36$ then $\sqrt{36} = 6$

$\sqrt{100}$ asks "What positive number **times itself** equals 100. Since $10 \bullet 10 = 100$ then $\sqrt{100} = 10$

$\sqrt{\frac{16}{81}}$ asks "What positive number **times itself** equals $\frac{16}{81}$ Since $\frac{4}{9} \bullet \frac{4}{9} = \frac{16}{81}$ then $\sqrt{\frac{16}{81}} = \frac{4}{9}$

Example 1

Find $\sqrt{9}$

$3 \bullet 3 = 9$ so

$\sqrt{9} = 3$

Example 2

Find $\sqrt{25}$

$5 \bullet 5 = 25$ so

$\sqrt{25} = 5$

Example 3

Find $\sqrt{81}$

$9 \bullet 9 = 81$ so

$\sqrt{81} = 9$

Example 4

Find $\sqrt{169}$

$13 \bullet 13 = 169$ so

$\sqrt{169} = 13$

Example 5

Find $\sqrt{4}$

$2 \bullet 2 = 4$ so

$\sqrt{4} = 2$

Example 6

Find $\sqrt{144}$

$12 \bullet 12 = 144$ so

$\sqrt{144} = 12$

Example 7

Find $\sqrt{\frac{36}{49}}$

$\frac{6}{7} \bullet \frac{6}{7} = \frac{36}{49}$ so

$\sqrt{\frac{36}{49}} = \frac{6}{7}$

Example 8

Find $\sqrt{\frac{100}{121}}$

$\frac{10}{11} \bullet \frac{10}{11} = \frac{100}{121}$ so

$\sqrt{\frac{100}{121}} = \frac{10}{11}$

Example 9

Find $\sqrt{\frac{18}{50}}$

$\sqrt{\frac{18}{50}} = \sqrt{\frac{9}{25}}$

$\sqrt{\frac{9}{25}} = \frac{3}{5}$

Addition, Subtraction Involving Square Roots

The operations of Addition and Subtraction can be applied to problems involving square roots. In the examples below **you CANNOT add the square roots together.** You must first reduce each square root to a whole number and then combine the numbers.

Example 1

$\sqrt{25}+\sqrt{49}$

first simplify each square root

$=5+7$

$=12$

Example 2

$\sqrt{36}+\sqrt{4}$

first simplify each square root

$=6+2$

$=8$

Example 3

$\sqrt{81}-\sqrt{16}$

first simplify each square root

$=9-4$

$=5$

Example 3

$\sqrt{25}-\sqrt{100}$

first simplify each square root

$=5-10$

$=-5$

Multiplying a Constant times a Square Root

A constant in front of a square root means the square root has been **multiplied** by the constant. **You CANNOT multiply the constant and the square root together.** You must first reduce the square root and then multiply the numbers.

Example 4

Simplify $5\sqrt{9}$

$5\sqrt{9}$ means

5 times $\sqrt{9}$

$5\bullet\sqrt{9}$

$=5\bullet 3$

$=15$

Example 5

Simplify $-3\sqrt{49}$

$-3\sqrt{49}$ means

-3 times $\sqrt{49}$

$-3\bullet\sqrt{49}$

$=-3\bullet 7$

$=-21$

Example 6

Simplify $-\sqrt{81}$

$-\sqrt{81}$ means

-1 times $\sqrt{81}$

$-1\bullet\sqrt{81}$

$=-1\bullet 9$

$=-9$

Example 7

Simplify $8\sqrt{\frac{25}{4}}$

$8\sqrt{\frac{25}{4}}$ means $8\bullet\sqrt{\frac{25}{4}}$

$=8\bullet\frac{5}{2}$

$=20$

Example 8

Simplify $-12\sqrt{\frac{9}{16}}$

$-12\sqrt{\frac{9}{16}}$ means $-12\bullet\sqrt{\frac{9}{16}}$

$=-12\bullet\frac{3}{4}$

$=-9$

Example 9

Simplify $-15\sqrt{\frac{49}{9}}$

$-15\sqrt{\frac{49}{9}}$ means $-15\bullet\sqrt{\frac{49}{9}}$

$=-15\bullet\frac{7}{3}$

$=-35$

Example 10

Simplify $3\sqrt{100}+2\sqrt{81}$

$$3\sqrt{100}+2\sqrt{81}$$
$$=3(10)+2(9)$$
$$=30+18$$
$$=48$$

Example 11

Simplify $-3\sqrt{16}-\sqrt{25}$

$$-3\sqrt{16}-\sqrt{25}$$
$$=-3(4)-(5)$$
$$=-12-5$$
$$=-17$$

Example 12

Simplify $8\sqrt{\frac{9}{16}}-12\sqrt{\frac{25}{9}}$

$$8\sqrt{\frac{9}{16}}-12\sqrt{\frac{25}{9}}$$
$$=8\left(\frac{3}{4}\right)-12\left(\frac{5}{3}\right)$$
$$=6-20$$
$$=-14$$

