

Section 8 – 5:

Solving Rational Equations

Solving Equations with Fractions

In section 2–3 we solved equations that had fractions with whole numbers in the denominators. In that section we solved the equations by first using the multiplication property of equality to multiply **every term in an equation** by the Least Common Denominator. When we multiply each term by the LCD the resulting equation will not have any fractions in it.

Example

$$\frac{4x}{5} + \frac{3}{2} = \frac{3x}{10} \quad \text{multiply all 3 terms by 10}$$

$$\left(\frac{10}{1}\right)\frac{4x}{5} + \left(\frac{10}{1}\right)\frac{3}{2} = \left(\frac{10}{1}\right)\frac{3x}{10}$$

$$8x + 15 = 3x \quad \text{subtract } 3x \text{ from both sides}$$
$$-3x \quad -3x$$

$$5x + 15 = 0 \quad \text{subtract 15 from both sides}$$

$$\frac{5x}{5} = \frac{-15}{5} \quad (\text{divide both sides by 5})$$

$$x = -3$$

Solving Rational Equations

All of the **Rational Equations** in this section will have some denominators that contain polynomials.

$$\frac{1}{6} - \frac{x-5}{3x} = \frac{x+5}{4x}$$

$$\frac{1}{x(x-1)} - \frac{x-5}{x} = \frac{x+5}{x-1}$$

When we solve the rational equations in this section we will use the same method as before. We start by multiplying **every term in an equation** by the Least Common Denominator. The difference in this section is that the LCD will not be a whole number as it was in section 2–3.

$$\frac{x-4}{2(x+2)} = \frac{5}{4}$$

The LCD is $4(x+2)$

$$\frac{2}{3x} = \frac{x}{6} + \frac{-4}{(x+4)}$$

The LCD is $6x(x+4)$

How to solve a Rational Equation

1. Factor each denominator if possible.
2. Find the LCD for all the denominators.
3. Multiply every term in the equation by the LCD. The denominator of each term will be canceled out in this process. This will leave you with an equation without fractions.
4. Solve the equation for the variable.
5. Check your solution. Plug the value for x into the expression in place of the variable. If a solution makes any of the denominators of the original rational equation equal to zero then that solution cannot be used and there is no solution or \emptyset .

Example 1

Solve for x

$$\frac{2}{3x} + \frac{1}{6} = \frac{3}{2x}$$

the LCD is $6x$ so
multiply each term of
the equation by $6x$

$$6x \left[\frac{2}{3x} + \frac{1}{6} = \frac{3}{2x} \right]$$

$$6x \cdot \frac{2}{3x} + 6x \cdot \frac{1}{6} = 6x \cdot \frac{3}{2x}$$

$$6^2 x \cdot \frac{2}{3x} + 6x \cdot \frac{1}{6} = 6^3 x \cdot \frac{3}{2^1 x}$$

$$4 + x = 9$$

$$x = 5$$

(5 does not cause a 0 in the denominator so)

$$x = 5$$

Example 2

Solve for x

$$\frac{1}{6x} - \frac{1}{4} = \frac{-1}{3x}$$

the LCD is $12x$ so
multiply each term of
the equation by $12x$

$$12x \left[\frac{1}{6x} - \frac{1}{4} = \frac{-1}{3x} \right]$$

$$12x \cdot \frac{1}{6x} - 12x \cdot \frac{1}{4} = 12x \cdot \frac{-1}{3x}$$

$$12^2 x \cdot \frac{1}{6x} - 12^3 x \cdot \frac{1}{4} = 12^4 x \cdot \frac{-1}{3x}$$

$$2 - 3x = -4$$

$$-3x = -6$$

$$x = 2$$

(2 does not cause a 0 in the denominator so)

$$x = 2$$

Example 3

Solve for x

$$\frac{x-4}{3x} + \frac{1}{2} = \frac{3}{4x}$$

the LCD is $12x$ so
multiply each term of
the equation by $6x$

$$12x \left[\frac{x-4}{3x} + \frac{1}{2} = \frac{3}{4x} \right]$$

$$12x \cdot \frac{x-4}{3x} + 12x \cdot \frac{1}{2} = 12x \cdot \frac{3}{4x}$$

$$12^4 x \cdot \frac{x-4}{3x} + 12^6 x \cdot \frac{1}{2} = 12^3 x \cdot \frac{3}{4x}$$

$$4(x-4) + 6x = 9$$

$$4x - 16 + 6x = 9$$

$$10x = 25$$

$$x = \frac{25}{10} = \frac{5}{2}$$

$\frac{5}{2}$ does not cause a 0 in the denominator so

$$x = \frac{5}{2}$$

Example 4

Solve for x

$$\frac{1}{6} - \frac{x-5}{3x} = \frac{x+5}{4x}$$

the LCD is $12x$ so
multiply each term of
the equation by $12x$

$$12x \left[\frac{1}{6} - \frac{x-5}{3x} = \frac{x+5}{4x} \right]$$

$$12x \cdot \frac{1}{6} - 12x \cdot \frac{x-5}{3x} = 12x \cdot \frac{x+5}{4x}$$

$$12^2 x \cdot \frac{1}{6} - 12^4 x \cdot \frac{x-5}{3x} = 12^3 x \cdot \frac{x+5}{4x}$$

$$2x - 4(x-5) = 3(x+5)$$

$$2x - 4x + 20 = 3x + 15$$

$$-2x + 20 = 3x + 15$$

$$5 = 5x$$

$$1 = x$$

1 does not cause a 0 in the denominator so

$$1 = x$$

Example 5

Solve for x

$$\frac{x}{x-2} - 4 = \frac{2}{x-2}$$

the LCD is $(x-2)$ so
multiply each term of
the equation by $(x-2)$

$$(x-2) \left[\frac{x}{x-2} - 4 = \frac{2}{x-2} \right]$$

$$\cancel{(x-2)} \cdot \frac{x}{\cancel{x-2}} - (x-2) \cdot 4 = \cancel{(x-2)} \cdot \frac{2}{\cancel{x-2}}$$

$$x - 4(x-2) = 2$$

$$x - 4x + 8 = 2$$

$$-3x = -6$$

$$x = 2$$

2 does cause a 0 in the denominator so

\emptyset or No Solution

Example 6

Solve for x

$$\frac{x-6}{x(x-3)} = \frac{-2}{2(x-3)}$$

the LCD is $2x(x-3)$ so
multiply each term of
the equation by $2x(x-3)$

$$2x(x-3) \left[\frac{x-6}{x(x-3)} = \frac{-2}{2(x-3)} \right]$$

$$2 \cancel{x} \cancel{(x-3)} \cdot \frac{x-6}{\cancel{x} \cancel{(x-3)}} = 2 \cancel{x} \cancel{(x-3)} \cdot \frac{-2}{2 \cancel{(x-3)}}$$

$$2(x-6) = -2x$$

$$2x - 12 = -2x$$

$$4x = 12$$

$$x = 3$$

3 does cause a 0 in the denominator so

\emptyset or No Solution

Example 7

Solve for x

$$\frac{3}{x+1} = \frac{1}{x-1} - \frac{2}{x^2-1}$$

factor each denominator

$$\frac{3}{x+1} = \frac{1}{x-1} - \frac{2}{(x+1)(x-1)}$$

the LCD is $(x+1)(x-1)$ so

multiply each term of

the equation by $(x+1)(x-1)$

$$(x+1)(x-1) \left[\frac{3}{x+1} = \frac{1}{x-1} - \frac{2}{(x+1)(x-1)} \right]$$

$$\cancel{(x+1)}(x-1) \cdot \frac{3}{\cancel{x+1}} = (x+1) \cancel{(x-1)} \cdot \frac{1}{\cancel{x-1}} - \cancel{(x+1)} \cancel{(x-1)} \cdot \frac{2}{\cancel{(x+1)} \cancel{(x-1)}}$$

$$3(x-1) = 1(x+1) - 2$$

$$3x - 3 = x + 1 - 2$$

$$3x - 3 = x - 1$$

$$2x = 2$$

$$x = 1$$

1 causes a 0 in the denominator so

No Solution or \emptyset

Example 8

Solve for x

$$\frac{2x+3}{3x-9} - \frac{x}{2x-6} = \frac{4}{3}$$

factor each denominator

$$\frac{2x+3}{3(x-3)} - \frac{x}{2(x-3)} = \frac{4}{3}$$

the LCD is $6(x-3)$ so

multiply each term of

the equation by $6(x-3)$

$$6(x-3) \left[\frac{2x+3}{3(x-3)} - \frac{x}{2(x-3)} = \frac{4}{3} \right]$$

$$\cancel{6^2(x-3)} \cdot \frac{2x+3}{\cancel{3(x-3)}} - \cancel{6^3(x-3)} \cdot \frac{x}{\cancel{2(x-3)}} = \cancel{6^2(x-3)} \cdot \frac{4}{3}$$

$$2(2x+3) - 3x = 8(x-3)$$

$$4x+6-3x=8x-24$$

$$x+6=8x-24$$

$$30=7x$$

$$\frac{30}{7}=x$$

$\frac{30}{7}$ does cause a 0 in the denominator so

$$\frac{30}{7}=x$$