Section 8 – 4B: Adding and Subtracting Rational Expressions

Adding and Subtracting Fractions without a Common Denominator

To add or subtract fractions, each fraction **MUST have the same denominator**. If the fractions do not have a common denominator **you must make new fractions that both have a common denominator**. To make each fraction’s denominator into the Least Common Denominator (LCD), multiply the top and bottom of each fraction by the fraction equal to 1 that will cause each denominator to become the LCD.

Example 1

\[
\frac{7}{4} + \frac{5}{6}
\]

The LCD is 12

multiply \( \frac{7}{4} \) by \( \frac{3}{3} \) to get an LCD of \( \frac{21}{12} \)

multiply \( \frac{5}{6} \) by \( \frac{2}{2} \) to get an LCD of \( \frac{10}{12} \)

\[
\frac{3 \cdot 7}{3 \cdot 4} + \frac{2 \cdot 5}{2 \cdot 6}
\]

\[
\frac{21}{12} + \frac{10}{12}
\]

\[
\frac{21+10}{12} \quad \text{add the numerators over the common denominator}
\]

\[
\frac{31}{12} \quad \text{combine like terms in the numerator}
\]

\[
= \frac{31}{12}
\]
Example 2

\[
\frac{x - 1}{4x} + \frac{x + 2}{10x}
\]

The LCD is \(20x\)

multiply \(\frac{x - 1}{4x}\) by \(\frac{5}{5}\) to get an LCD of \(\frac{20x}{20x}\)

multiply \(\frac{x + 2}{10x}\) by \(\frac{2}{2}\) to get an LCD of \(\frac{20x}{20x}\)

\[
\frac{5}{5} \cdot \frac{(x - 1)}{4x} + \frac{2}{2} \cdot \frac{(x + 2)}{10x}
\]

\[
\frac{5}{4x} \cdot (x - 1) + \frac{2}{10x} \cdot (x + 2)
\]
distribute the 5 and the 2 in each numerator

\[
\frac{5}{4x} \cdot (x - 1) + \frac{2}{10x} \cdot (x + 2)
\]
multiply the 5 and 2 in each denominator

\[
\frac{5x - 5}{20x} + \frac{2x + 4}{20x}
\]

\[
= \frac{5x - 5 + (2x + 4)}{20x}
\]
add the numerators with the second one in brackets

\[
= \frac{5x - 5 + 2x + 4}{20x}
\]
distribute the +

\[
= \frac{7x - 1}{20x}
\]
combine like terms in the numerator

\[
= \frac{7x - 1}{20x}
\]
over the common denominator
Example 3

Add

\[
\frac{2x-1}{6(x+4)} + \frac{x-3}{4(x+4)}
\]

The LCD is \(12(x+4)\)

multiply \(\frac{2x-1}{6(x+4)}\) by \(\frac{2}{2}\) to get an LCD of \(\frac{12}{12(x+4)}\)

multiply \(\frac{x-3}{4(x+4)}\) by \(\frac{3}{3}\) to get an LCD of \(\frac{12}{12(x+4)}\)

\[
\frac{2}{2} \cdot \frac{2x-1}{6(x+4)} + \frac{3}{3} \cdot \frac{x-3}{4(x+4)}
\]

\[
\frac{2}{2} \cdot \frac{2x-1}{6(x+4)} + \frac{3}{3} \cdot \frac{x-3}{4(x+4)} \quad \text{distribute the 2 and the 3 in each numerator}
\]

\[
\frac{2}{2} \cdot \frac{2x-1}{6(x+4)} + \frac{3}{3} \cdot \frac{x-3}{4(x+4)} \quad \text{multiply the 2 and 3 in each denominator}
\]

\[
\frac{4x-2}{12(x+4)} + \frac{3x-9}{12(x+4)}
\]

\[
= \frac{4x-2 + (3x-9)}{12(x+4)} \quad \text{add the numerators with the second one in brackets over the common denominator}
\]

\[
= \frac{4x-2 + 3x-9}{12(x+4)} \quad \text{distribute the +}
\]

\[
= \frac{4x-2 + 3x-9}{12(x+4)} \quad \text{combine like terms in the numerator over the common denominator}
\]

\[
= \frac{7x-11}{12(x+4)}
\]
Example 4

Add

\[
\frac{6}{x-2} + \frac{3x+5}{x^2-4}
\]

factor each denominator

\[
\frac{6}{x-2} + \frac{3x+5}{(x-2)(x+2)}
\]

The LCD is \((x-2)(x+2)\)

multiply \(\frac{6}{x-2}\) by \(\frac{(x+2)}{(x+2)}\) to get an LCD of \(\frac{6(x+2)}{(x-2)(x+2)}\)

multiply \(\frac{3x+5}{(x-2)(x+2)}\) by \(\frac{1}{1}\) to get an LCD of \(\frac{3x+5}{(x-2)(x+2)}\)

\[
= \frac{(x+2)}{(x+2)} \cdot \frac{6}{(x-2)} + \frac{1}{1} \cdot \frac{3x+5}{(x-2)(x+2)}
\]

\[
= \frac{6x+12}{(x-2)(x+2)} + \frac{3x+5}{(x-2)(x+2)}
\]

\[
= \frac{6x+12 + 3x+5}{(x-1)(x+2)}
\]

add the numerators over the common denominator

combine like terms in the numerator

\[
= \frac{9x+17}{(x-1)(x+2)}
\]
Example 5

Add

\[ \frac{x + 3}{4x - 20} + \frac{5}{12} \]

factor each denominator

\[ \frac{x + 3}{4(x - 5)} + \frac{5}{12} \]

The LCD is \(12(x - 5)\)

multiply \(\frac{x + 3}{4(x - 5)}\) by \(\frac{3}{3}\) to get an LCD of \(\frac{12}{12(x - 5)}\)

multiply \(\frac{5}{12}\) by \(\frac{(x - 5)}{(x - 5)}\) to get an LCD of \(\frac{12}{12(x - 5)}\)

\[
\frac{3}{3} \cdot \frac{x + 3}{4(x - 5)} + \frac{5}{12} \cdot \frac{(x - 5)}{(x - 5)}
\]

\[
\frac{3x + 9}{12(x - 5)} + \frac{5x - 25}{12(x - 5)}
\]

\[
= \frac{3x + 9 + 5x - 25}{12(x - 5)} \quad \text{add the numerators over the common denominator}
\]

combine like terms in the numerator

\[
= \frac{8x - 16}{12(x - 5)}
\]

\[
= \frac{8(x - 2)}{12(x - 5)}
\]

\[
= \frac{8^2 (x - 2)}{12^2(x - 5)} = \frac{2 (x - 2)}{3(x - 5)}
\]
Example 6

Subtract

\[
\frac{2x - 1}{9(x + 4)} - \frac{3x - 2}{6(x + 4)}
\]

The LCD is \(18(x + 4)\)

multiply \(\frac{2x - 1}{9(x + 4)}\) by \(\frac{2}{2}\) to get an LCD of \(\frac{18(x + 4)}{}\)

multiply \(\frac{3x - 2}{6(x + 4)}\) by \(\frac{3}{3}\) to get an LCD of \(\frac{18(x + 4)}{}\)

\[
\frac{2}{2} \cdot \frac{2x - 1}{9(x + 4)} - \frac{3}{3} \cdot \frac{3x - 2}{6(x + 4)}
\]

\[
\frac{4x - 2}{18(x + 4)} - \frac{9x - 6}{18(x + 4)}
\]

\[
= \frac{4x - 2 - (9x - 6)}{18(x + 4)} \text{ subtract the numerators over the LCD}
\]

\[
\frac{4x - 2 - 9x + 6}{18(x + 4)} \text{ distrubte the } -
\]

\[
\frac{4x - 2 - 9x + 6}{18(x + 4)}
\]

combine like terms in the numerator

\[
= \frac{-5x + 4}{18(x + 4)}
\]
Example 7

Subtract

\[
\frac{4}{3x - 6} - \frac{2}{3x}
\]

factor each denominator

\[
\frac{4}{3(x - 2)} - \frac{2}{3x}
\]

The LCD is \(3x(x - 2)\)

multiply \(\frac{4}{3(x - 2)}\) by \(\frac{x}{x}\) to get an LCD of \(\frac{4x}{3x(x - 2)}\)

multiply \(\frac{2}{3x}\) by \(\frac{(x - 2)}{(x - 2)}\) to get an LCD of \(\frac{2x - 4}{3x(x - 2)}\)

\[
\frac{x}{x} \cdot \frac{4}{3(x - 2)} - \frac{2}{3x} \cdot \frac{(x - 2)}{(x - 2)}
\]

\[
\frac{4x}{3x(x - 2)} - \frac{2x - 4}{3x(x - 2)}
\]

\[
= \frac{4x - (2x - 4)}{3x(x - 2)}
\]

subtract the numerators over the LCD

\[
= \frac{4x - (2x - 4)}{3x(x - 2)}
\]

\[
= \frac{4x - 2x + 4}{3x(x - 2)}
\]

combine like terms in the numerator

\[
= \frac{2x + 4}{3x(x - 2)}
\]
Example 8

Subtract

\[ \frac{4}{x+1} - \frac{2}{x-2} \]

The LCD is \((x-1)(x-2)\)

multiply \(\frac{4}{x+1}\) by \(\frac{x-2}{x-2}\) to get an LCD of \(\frac{(x-1)(x-2)}{(x+1)(x-2)}\)

multiply \(\frac{2}{x-2}\) by \(\frac{x+1}{x+1}\) to get an LCD of \(\frac{(x+1)(x-2)}{(x+1)(x-2)}\)

\[ \frac{(x-2)}{(x-2)} \cdot \frac{4}{x+1} - \frac{(x+1)}{(x+1)} \cdot \frac{2}{x-2} \]

\[ \frac{4 - 8x}{(x+1)(x-2)} - \frac{2x + 2}{(x+1)(x-2)} \]

\[ = \frac{4x - 8 - (2x + 2)}{(x+1)(x-2)} \] subtract the numerators over the LCD

distribute the –

\[ \frac{4x - 8 - 2x - 2}{(x+1)(x-2)} \]

combine like terms in the numerator

\[ = \frac{2x - 10}{(x+1)(x-2)} \] Factor the numerator

\[ = \frac{2(x-5)}{(x+1)(x-2)} \] Does not reduce
Example 9

Subtract

\[ \frac{4}{3x+3} - \frac{x-4}{2x^2-2} \]

factor each denominator

\[ \frac{4}{3(x+1)} - \frac{x-4}{2(x-1)(x+1)} \]

The LCD is \(6(x+1)(x-1)\)

multiply \( \frac{4}{3(x+1)} \) by \( \frac{2(x-1)}{2(x-1)} \) to get an LCD of \( \frac{6(x+1)(x-1)}{6(x+1)(x-1)} \)

multiply \( \frac{x-4}{2(x-1)(x+1)} \) by \( \frac{3}{3} \) to get an LCD of \( \frac{6(x+1)(x-1)}{6(x+1)(x-1)} \)

\[ \frac{2(x-1)}{2(x-1)} \cdot \frac{4}{3(x+1)} - \frac{3}{3} \cdot \frac{x-4}{2(x-1)(x+1)} \]

\[ \frac{8x-8}{6(x+1)(x-1)} - \frac{3x-12}{6(x+1)(x-1)} \]

\[ = \frac{8x-8-(3x-12)}{6(x+1)(x-1)} \] subtract the numerators over the LCD

\[ = \frac{8x-8-(3x-12)}{6(x+1)(x-1)} \] distrubte the –

\[ = \frac{8x-8-3x+12}{6(x+1)(x-1)} \]

combine like terms in the numerator

\[ = \frac{5x+4}{6(x+1)(x-1)} \]