

## Section 8 – 4B: Adding and Subtracting Rational Expressions

### Adding and Subtracting Fractions without a Common Denominator

To add or subtract fractions, each fraction **MUST have the same denominator**. If the fractions do not have a common denominator **you must make new fractions that both have a common denominator**. To make each fraction's denominator into the Least Common Denominator (LCD), multiply the top and bottom of each fraction by the fraction equal to 1 that will cause each denominator to become the LCD.

#### Example 1

$$\frac{7}{4} + \frac{5}{6}$$

The LCD is 12

multiply  $\frac{7}{4}$  by  $\frac{3}{3}$  to get an LCD of  $\frac{\quad}{12}$

multiply  $\frac{5}{6}$  by  $\frac{2}{2}$  to get an LCD of  $\frac{\quad}{12}$

$$\frac{3}{3} \cdot \frac{7}{4} + \frac{2}{2} \cdot \frac{5}{6}$$

$$\frac{21}{12} + \frac{10}{12}$$

$$= \frac{21+10}{12} \quad \begin{array}{l} \text{add the numerators} \\ \text{over the common denominator} \end{array}$$

$$= \frac{31}{12} \quad \text{combine like terms in the numerator}$$

$$= \frac{31}{12}$$

## Example 2

$$\frac{x-1}{4x} + \frac{x+2}{10x}$$

The LCD is  $20x$

multiply  $\frac{x-1}{4x}$  by  $\frac{5}{5}$  to get an LCD of  $\frac{\quad}{20x}$

multiply  $\frac{x+2}{10x}$  by  $\frac{2}{2}$  to get an LCD of  $\frac{\quad}{20x}$

$$\frac{5 \cdot (x-1)}{5 \cdot 4x} + \frac{2 \cdot (x+2)}{2 \cdot 10x}$$

$$\frac{5 \cdot (x-1)}{5 \cdot 4x} + \frac{2 \cdot (x+2)}{2 \cdot 10x} \quad \begin{array}{l} \text{distribute the 5 and the 2 in each numerator} \\ \text{multiply the 5 and 2 in each denominator} \end{array}$$

$$\frac{5x-5}{20x} + \frac{2x+4}{20x}$$

$$= \frac{5x-5+(2x+4)}{20x} \quad \begin{array}{l} \text{add the numerators with the second one in brackets} \\ \text{over the common denominator} \end{array}$$

$$= \frac{5x-5+2x+4}{20x} \quad \text{distribute the +}$$

$$= \frac{7x-1}{20x} \quad \begin{array}{l} \text{combine like terms in the numerator} \\ \text{over the common denominator} \end{array}$$

$$= \frac{7x-1}{20x}$$

### Example 3

**Add**

$$\frac{2x-1}{6(x+4)} + \frac{x-3}{4(x+4)}$$

The LCD is  $12(x+4)$

multiply  $\frac{2x-1}{6(x+4)}$  by  $\frac{2}{2}$  to get an LCD of  $\frac{\quad}{12(x+4)}$

multiply  $\frac{x-3}{4(x+4)}$  by  $\frac{3}{3}$  to get an LCD of  $\frac{\quad}{12(x+4)}$

$$\frac{2}{2} \cdot \frac{2x-1}{6(x+4)} + \frac{3}{3} \cdot \frac{x-3}{4(x+4)}$$

$$\frac{2}{2} \cdot \frac{(2x-1)}{6(x+4)} + \frac{3}{3} \cdot \frac{(x-3)}{4(x+4)}$$

distribute the 2 and the 3 in each numerator  
multiply the 2 and 3 in each denominator

$$\frac{4x-2}{12(x+4)} + \frac{3x-9}{12(x+4)}$$

$$= \frac{4x-2+(3x-9)}{12(x+4)}$$

add the numerators with the second one in brackets  
over the common denominator

$$= \frac{4x-2+3x-9}{12(x+4)}$$

distribute the +

$$= \frac{4x-2+3x-9}{12(x+4)}$$

combine like terms in the numerator  
over the common denominator

$$= \frac{7x-11}{12(x+4)}$$

### Example 4

**Add**

$$\frac{6}{x-2} + \frac{3x+5}{x^2-4} \quad \text{factor each denominator}$$

$$\frac{6}{(x-2)} + \frac{3x+5}{(x-2)(x+2)}$$

The LCD is  $(x-2)(x+2)$

$$\text{multiply } \frac{6}{(x-2)} \text{ by } \frac{(x+2)}{(x+2)} \text{ to get an LCD of } \frac{\quad}{(x-2)(x+2)}$$

$$\text{multiply } \frac{3x+5}{(x-2)(x+2)} \text{ by } \frac{1}{1} \text{ to get an LCD of } \frac{\quad}{(x-2)(x+2)}$$

$$= \frac{(x+2)}{(x+2)} \cdot \frac{6}{(x-2)} + \frac{1}{1} \cdot \frac{3x+5}{(x-2)(x+2)}$$

$$= \frac{6x+12}{(x-2)(x+2)} + \frac{3x+5}{(x-2)(x+2)}$$

$$= \frac{6x+12+3x+5}{(x-2)(x+2)} \quad \begin{array}{l} \text{add the numerators} \\ \text{over the common denominator} \end{array}$$

combine like terms in the numerator

$$= \frac{9x+17}{(x-2)(x+2)}$$

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### Example 5

**Add**

$$\frac{x+3}{4x-20} + \frac{5}{12} \text{ factor each denominator}$$

$$\frac{x+3}{4(x-5)} + \frac{5}{12}$$

The LCD is  $12(x-5)$

multiply  $\frac{x+3}{4(x-5)}$  by  $\frac{3}{3}$  to get an LCD of  $\frac{\quad}{12(x-5)}$

multiply  $\frac{5}{12}$  by  $\frac{(x-5)}{(x-5)}$  to get an LCD of  $\frac{\quad}{12(x-5)}$

$$\frac{3}{3} \cdot \frac{x+3}{4(x-5)} + \frac{(x-5)}{(x-5)} \cdot \frac{5}{12}$$

$$\frac{3x+9}{12(x-5)} + \frac{5x-25}{12(x-5)}$$

$$= \frac{3x+9+5x-25}{12(x-5)} \quad \begin{array}{l} \text{add the numerators} \\ \text{over the common denominator} \end{array}$$

combine like terms in the numerator

$$= \frac{8x-16}{12(x-5)}$$

$$= \frac{8(x-2)}{12(x-5)}$$

$$= \frac{8^2(x-2)}{12^3(x-5)} = \frac{2(x-2)}{3(x-5)}$$

## Example 6

### Subtract

$$\frac{2x-1}{9(x+4)} - \frac{3x-2}{6(x+4)}$$

The LCD is  $18(x+4)$

multiply  $\frac{2x-1}{9(x+4)}$  by  $\frac{2}{2}$  to get an LCD of  $\frac{\quad}{18(x+4)}$

multiply  $\frac{3x-2}{6(x+4)}$  by  $\frac{3}{3}$  to get an LCD of  $\frac{\quad}{18(x+4)}$

$$\frac{2}{2} \cdot \frac{2x-1}{9(x+4)} - \frac{3}{3} \cdot \frac{3x-2}{6(x+4)}$$

$$\frac{4x-2}{18(x+4)} - \frac{9x-6}{18(x+4)}$$

$$= \frac{4x-2-(9x-6)}{18(x+4)} \quad \begin{array}{l} \text{subtract the numerators} \\ \text{over the LCD} \end{array}$$

$$\frac{4x-2-(9x-6)}{18(x+4)} \quad \text{distribute the } -$$

$$\frac{4x-2-9x+6}{18(x+4)}$$

combine like terms in the numerator

$$= \frac{-5x+4}{18(x+4)}$$

## Example 7

### Subtract

$$\frac{4}{3x-6} - \frac{2}{3x} \text{ factor each denominator}$$

$$\frac{4}{3(x-2)} - \frac{2}{3x}$$

The LCD is  $3x(x-2)$

multiply  $\frac{4}{3(x-2)}$  by  $\frac{x}{x}$  to get an LCD of  $\frac{\quad}{3x(x-2)}$

multiply  $\frac{2}{3x}$  by  $\frac{(x-2)}{(x-2)}$  to get an LCD of  $\frac{\quad}{3x(x-2)}$

$$\frac{x}{x} \cdot \frac{4}{3(x-2)} - \frac{2}{3x} \cdot \frac{(x-2)}{(x-2)}$$

$$\frac{4x}{3x(x-2)} - \frac{2x-4}{3x(x-2)}$$

$$= \frac{4x - (2x - 4)}{3x(x-2)} \text{ subtract the numerators}$$

over the LCD

$$\frac{4x - (2x - 4)}{3x(x-2)} \text{ distribute the } -$$

$$\frac{4x - 2x + 8}{3x(x-2)}$$

combine like terms in the numerator

$$= \frac{2x + 8}{3x(x-2)}$$

## Example 8

### Subtract

$$\frac{4}{x+1} - \frac{2}{x-2}$$

The LCD is  $(x-1)(x-2)$

multiply  $\frac{4}{x+1}$  by  $\frac{(x-2)}{(x-2)}$  to get an LCD of  $\frac{\quad}{(x+1)(x-2)}$

multiply  $\frac{2}{x-2}$  by  $\frac{(x+1)}{(x+1)}$  to get an LCD of  $\frac{\quad}{(x+1)(x-2)}$

$$\frac{(x-2)}{(x-2)} \cdot \frac{4}{x+1} - \frac{(x+1)}{(x+1)} \cdot \frac{2}{x-2}$$

$$\frac{4-8x}{(x+1)(x-2)} - \frac{2x+2}{(x+1)(x-2)}$$

$$= \frac{4x-8-(2x+2)}{(x+1)(x-2)} \quad \begin{array}{l} \text{subtract the numerators} \\ \text{over the LCD} \end{array}$$

$$\frac{4x-8-(2x+2)}{(x+1)(x-2)} \quad \begin{array}{l} \text{distribute the } - \\ \end{array}$$

$$\frac{4x-8-2x-2}{(x+1)(x-2)}$$

combine like terms in the numerator

$$= \frac{2x-10}{(x+1)(x-2)} \quad \text{Factor the numerator}$$

$$= \frac{2(x-5)}{(x+1)(x-2)} \quad \text{Does not reduce}$$

## Example 9

### Subtract

$$\frac{4}{3x+3} - \frac{x-4}{2x^2-2} \quad \text{factor each denominator}$$

$$\frac{4}{3(x+1)} - \frac{x-4}{2(x-1)(x+1)}$$

The LCD is  $6(x+1)(x-1)$

$$\text{multiply } \frac{4}{3(x+1)} \text{ by } \frac{2(x-1)}{2(x-1)} \text{ to get an LCD of } \frac{\quad}{6(x+1)(x-1)}$$

$$\text{multiply } \frac{x-4}{2(x-1)(x+1)} \text{ by } \frac{3}{3} \text{ to get an LCD of } \frac{\quad}{6(x+1)(x-1)}$$

$$\frac{2(x-1)}{2(x-1)} \cdot \frac{4}{3(x+1)} - \frac{3}{3} \cdot \frac{x-4}{2(x-1)(x+1)}$$

$$\frac{8x-8}{6(x+1)(x-1)} - \frac{3x-12}{6(x+1)(x-1)}$$

$$= \frac{8x-8-(3x-12)}{6(x+1)(x-1)} \quad \begin{array}{l} \text{subtract the numerators} \\ \text{over the LCD} \end{array}$$

$$= \frac{8x-8-(3x-12)}{6(x+1)(x-1)} \quad \text{distribute the } -$$

$$= \frac{8x-8-3x+12}{6(x+1)(x-1)}$$

combine like terms in the numerator

$$= \frac{5x+4}{6(x+1)(x-1)}$$