

Section 8 – 3: Least Common Denominators

Before 2 two or more rational expressions (fractions) can be added or subtracted they must have a **common denominator**. The smallest of all the possible common denominators is called the **Least Common Denominator or LCD**. This section will help develop the skill of finding the LCD of rational expressions and the next section will develop the skill of adding and subtracting them.

How to find the Least Common Denominator (LCD) of two Rational Expressions

1. Factor each **Denominator**.
2. The LCD is a product of the LCD of the **coefficients** and the LCD of the **variable terms**.
 - 2a. The LCD for the coefficients is **the smallest number each coefficient divides into evenly**.
 - 2b. The LCD for the variable part of the denominator is the **product** of **each** of the **different factors** raised to the **highest power it appears** in the different denominators.

Take each different variable factor
raised to the **highest power it appears** in the different denominators.

Example 1

Find the LCD for

$$\frac{5}{4xy^3} \text{ and } \frac{7}{6x^2y}$$

1. $4 \cdot x \cdot y^2$ and $6 \cdot x^2 \cdot y$ are already factored
- 2A. The LCD of 4 and 6 is the smallest number 4 and 6 BOTH divide INTO
The LCD of 4 and 6 is 12
- 2B. the highest power of x is x^2
the highest power of y is y^3
3. The LCD of $4xy^3$ and $6x^2y$ is $12x^2y^3$

Example 2

Find the LCD for

$$\frac{x+1}{6(x+2)} \text{ and } \frac{x}{8x(x-1)}$$

1. $6(x+2)$ and $8(x-1)$ are already factored
- 2A. The LCD of 6 and 8 is 24
- 2B. The highest power of x is x
The highest power of $(x+2)$ is $(x+2)$
The highest power of $(x-1)$ is $(x-1)$
3. The LCD = $24x(x+2)(x-1)$

Example 3

Find the LCD for

$$\frac{7}{4x(x-3)} \text{ and } \frac{x-1}{5x^2(x-3)}$$

1. $4x(x-3)$ and $5x^2(x-3)$
are already factored
- 2A. The LCD of 4 and 5 is 20
- 2B. the highest power of x and x^2 is x^2
the highest power of $(x-3)$ is $(x-3)^1$
3. The LCD of $4x(x-3)$ and $5x^2(x-3)$
 $= 20x^2(x-3)$

Example 4

Find the LCD for

$$\frac{3}{9x^3(x-1)} \text{ and } \frac{2}{6x^2(x-1)^2}$$

1. $9x^2(x-1)$ and $6x(x-1)^2$
are already factored
- 2A. the LCD of 9 and 6 is 18
- 2B. The highest power of x is x^3
The highest power of $(x-1)$ is $(x-1)^2$
3. The LCD of $9x^3(x-1)$ and $6x^2(x-1)^2$
is $18x^3(x-1)^2$

Example 5

Find the LCD for

$$\frac{x}{3x+21} \text{ and } \frac{x-1}{4x+28}$$

1. $3x+21$ and $4x+28$ factor as
 $3(x+7)$ and the $4(x+7)$
- 2A. The LCD of 3 and 4 is 12
- 2B. The highest power of $(x+7)$ is $(x+7)$
3. The LCD is $12(x+7)$

Example 6

Find the LCD for

$$\frac{x}{x^2-9} \text{ and } \frac{x-1}{2x-6}$$

1. x^2-9 and $2x-6$ factor as
 $1(x+3)(x-3)$ and $2(x-3)$
- 2A. The LCD of 1 and 2 is 2
- 2B. The highest power of $(x+3)$ is $(x+3)$
The highest power of $(x-3)$ is $(x-3)$
3. The LCD is $2(x+3)(x-3)$

Example 7

Find the LCD for

$$\frac{-2}{5x^3 - 5x^2} \text{ and } \frac{4}{2x^2 - 2x}$$

1. $5x^3 - 5x^2$ and $2x^2 - 2x$ factor as

$$5x^2(x-1) \text{ and } 2x(x-1)$$

2A. The LCD of 5 and 2 is 10

- 2B. The highest power of x is x^2
The highest power of $(x-1)$ is $(x-1)$

3. The LCD = $10x^2(x-1)$

Example 9

Find the LCD for

$$\frac{5}{2x^3 - 6x^2} \text{ and } \frac{x-1}{4x-12}$$

1. $2x^3 - 6x^2$ and $4x - 12$ factor as
 $2x^2(x-3)$ and $4(x-3)$

2A. The LCD of 2 and 4 is 4

- 2B. The highest power of $(x-3)$ is $(x-3)$
The highest power of x is x^2

3. The LCD = $4x^2(x-3)$

Example 8

Find the LCD for

$$\frac{-x}{x^2 - 25} \text{ and } \frac{x-1}{x^2 + 10x + 25}$$

1. $x^2 - 25$ and $x^2 + 10x + 25$ factor as

$$(x-5)(x+5) \text{ and } (x+5)^2$$

2A. The LCD of 1 and 1 is 1

- 2B. The highest power of $(x-5)$ is $(x-5)$
The highest power of $(x+5)$ is $(x+5)^2$

3. The LCD is $(x+5)(x+5)^2$

Example 10

Find the LCD for

$$\frac{3x}{x^2 - 1} \text{ and } \frac{5}{3x + 3}$$

1. $x^2 - 1$ and $3x + 3$ factor as
 $1(x+1)(x-1)$ and $3(x+1)$

2A. The LCD of 1 and 3 is 3

- 2B. The highest power of $(x+1)$ is $(x+1)$
The highest power of $(x-1)$ is $(x-1)$

3. The LCD = $3(x+1)(x-1)$

Example 11

Find the LCD for

$$\frac{x}{x^2 + 4x + 4} \text{ and } \frac{2}{x^2 - 4}$$

1. $x^2 + 4x + 4$ and $x^2 - 4$ factor as
 $(x + 2)^2$ and $(x + 2)(x - 2)$
- 2A. The LCD of 1 and 1 is 1
- 2B. The highest power of $(x + 2)$ is $(x + 2)^2$
The highest power of $(x - 2)$ is $(x - 2)$
3. The LCD = $(x + 2)^2(x - 2)$

Example 12

Find the LCD for

$$\frac{7x}{5x^2 - 5} \text{ and } \frac{x + 7}{x^2 - 2x + 1}$$

1. $5x^2 - 5$ and $x^2 - 2x + 1$ factor as
 $5(x + 1)(x - 1)$ and $(x - 1)^2$
- 2A. The LCD of 5 and 1 is 5
- 2B. The highest power of $(x + 1)$ is $(x + 1)$
The highest power of $(x - 1)$ is $(x - 1)^2$
3. The LCD = $5(x + 1)(x - 1)^2$